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Timing flexibility and optimal repurchase volume of open market share repurchases

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Timing Flexibility and Optimal Repurchase Volume of Open Market Share Repurchases

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Abstract

Open market share repurchases (OMRs) are a tool companies use to distribute cash to shareholders (similar to dividends), adjust their capital structure, use as a signaling device and a tool to reduce moral hazard, and reduce their tax liabilities. They can also be used as an investment by the company itself in which it buys back its own (undervalued) shares and thus makes a profit between the price paid and the fundamental value of the shares. While there is a considerable literature on most of these aspects, not much work has been conducted on the optimal timing of share repurchases as an investment tool for the company.

Given that the conduct of share repurchases has a large number of flexibilities which are at the discretion of the company and its managers, any model of share repurchases has to adequately address them. In this thesis a real options approach has been used to model share repurchases, which allows to include many of the flexibilities share repurchases offer. We investigate the optimal timing of share repurchases through a model in which once started, the repurchase is conducted in a single time period. This model is then subsequently extended to allow the repurchase program to be suspended and restarted in a multi-time period setting. We also derive the optimal repurchase amount in both cases.

We derive conditions on the difference between the fundamental value and price that would trigger the initiation of share repurchases for both models and analyze the dependencies of this trigger value on a number of variables such as the volatility of the shares. An empirical analysis of the share repurchases conducted by Taiwanese IT companies show broad support for the model developed in this thesis. However, the small sample size does not allow us to derive definitive conclusions and a larger sample needs to be investigated in future research.

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I would like to dedicate this thesis to my father who has recently gone to heaven.

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Chapter 1
Introduction

1.1 Definition and motivation of share repurchases

Share repurchases are a mechanism by which companies return capital to their shareholders. The company acquires shares from existing shareholders in exchange for cash payment. They can either cancel the acquired shares and thus reduce their outstanding equity or retain the shares with the aim to redistribute them to new shareholders, e.g. as part of executive compensation plans or pension plans.

A corporation has many motives for repurchasing its shares. The most important goals usually mentioned in the academic literature and by managers are as follows:

1. *Substitution for dividends*. Similar to dividends, share repurchases distribute cash to shareholders. The main difference between share repurchases and dividends is that with share repurchases, not all shareholders obtain this cash but only those who sell their shares to the company. The remaining shareholders benefit indirectly as the future cash flow of the company is divided among fewer shares which should increase the value of the remaining shares (Woods/Brigham, 1966; Ofer/Thakor, 1987).

2. *Employee compensation.* Companies routinely pay part of the compensation to senior managers in the form of stock options or as shares rather than use cash payments. The aim of these compensation plans is to align the interests of the managers with that of the company, thereby reducing the moral hazard problem. (Weisbenner, 2001; Jolls, 1998). The shares which are acquired in share repurchases can be used either for distribution to the beneficiaries of these compensation plans or are accumulated to achieve a sufficient number of shares

in relation to the corresponding stock options which may be exercised by managers holding such options.

3. *Adjustment of the capital structure.* If shares are cancelled after they have been acquired by the company, this reduces the outstanding equity the company holds. Share repurchases can therefore be used as a tool to adjust the capital structure of the company. A similar result could obviously be achieved through the distribution of dividends (Leland/Pyle, 1977; Kim *et al.*, 2003).

4. *Takeover defence.* A company may conduct a share repurchase as a measure to reduce the likelihood of an unfriendly takeover approach. This not only reduces the amount of cash held by the company, making it a less attractive target, but it also enables the company to reduce the number of shares an acquirer could easily buy from the open market. If the share repurchase is conducted such that any "friendly" shareholder, i.e. those who would not willingly sell his shares to an acquirer, are not selling their shares, while the non-strategic shareholders sell their shares, this would serve to increase the fraction of friendly shareholders, hence making a takeover approach less likely to succeed (Vermaelen,1984).

5. *Signalling motives.* Managers may have better information than shareholder on the future prospects of the company. The decisions of managers can reveal this information to the market, thus acting as a signal; stock repurchases are one such signal which managers can use. The benefit to the manager and the company is that the stock price will react to this signal and adjust at least partially to the fundamental value.

6. *Investment activity.* Companies can also see their own stocks as investment vehicles, as much as any other securities they might invest in. Given the potential informational advantage the company has regarding the value of their own shares, share repurchases could be regarded as an attractive investment in cases where the market undervalues the shares (Vermaelen, 1981 and 1984; Maxwell/Stephens, 2003).

7. *Support for the share price.* Companies conducting share repurchases increase temporarily the demand for the shares, causing the share price to rise. Thus a company could use share repurchases as a tool to prevent a further fall of the share price, e.g. to avoid a violation of minimum equity requirements or to

prevent a takeover bid due to the low value of the shares (Ikenbery/Vermaelen, 1996).

8. *Tax reduction.* With dividends and capital gains taxed at different tax rates in some countries it can be more beneficial for some shareholders to sell their shares as part of a share repurchase programme of a company than to receive the same amount in dividends. In some countries where retained earnings are taxed at a lower rate than dividends, there are tax benefits to companies which conduct share repurchases. Share repurchases when used in this manner is a tax efficient way of distributing cash to shareholders (Woods/Brigham, 1996; Grullen/Michaely 2000).

1.2 Research aim

The aim of this thesis is to develop a method by which managers can decide on the optimal timing of a share repurchase. In doing this, we will focus exclusively on share repurchases as an investment activity by the company; any other motivations and complications which arise from share repurchases will be ignored.

As will be pointed out in chapter 2, companies have to announce their intention to conduct share repurchases well in advance of their commencement. As making such an announcement does not require the company to conduct the share repurchase, we can interpret it as the creation of an option to conduct share repurchases. The actual share repurchase occurs only upon the exercise of such an option. Using this idea, we will establish a framework using real options analysis to determine the conditions under which a company should exercise the option, i.e. conduct share repurchases. In using this approach, we follow the footsteps of the work by Ikenberry/Vermaelen (1996) but expand on their model in various ways.

The results derived in this thesis provide clear guidance to companies on the optimal timing of share repurchases and allows us to compare our theoretical results with the actual behaviour of companies in an empirical investigation. This investigation allows us to evaluate the relevance of our model for explaining actual share repurchases.

This thesis thus furthers our understanding of the optimal conduct of share repurchases and provides the first detailed study of share repurchases using real options analysis. When combining our results with that of the well established theories on share repurchases, it is possible to obtain a more complete picture of the motivation for conducting share repurchases.

1.3 Outline of the thesis

This thesis is organized into seven chapters. *Chapter 1* is an introduction. The regulatory framework governing share repurchases in a number of countries is set out in *chapter 2.* This chapter establishes the main characteristics of share repurchase activities that justify the use of a real options approach for further analysis.

Chapter 3 will provide a review of the established theories and empirical evidence for share repurchases. We will discuss a number of basic theories and the empirical evidence supporting them as well as provide a critical analysis of their shortcomings.

We will provide a brief introduction to real options theory in *chapter 4.* By showing that the regulatory framework set out in *chapter 2* can be well interpreted in a real options framework, we provide a detailed justification for our modelling approach. The chapter will close with a brief overview of some recent applications of real options analysis for corporate finance decisions, including the few contributions addressing share repurchases in such a framework.

Chapter 5 develops a model of share repurchases that is subsequently evaluated using real options analysis. We will derive the conditions under which share repurchases should be conducted and compare the results with those of models using different approaches when addressing the conduct of share repurchases.

By comparing the predictions of the models developed in *chapter 5* with actual share repurchases by companies in *chapter 6,* we evaluate the relevance of our models for the explanation of real world observed behaviour. Using these results, we can then assess whether our models can explain some of the empirical findings and how much the results are driven by other forces which we did not include in our model.

Finally, in *chapter 7,* we conclude our results and provide a critical analysis of the limitations of our model as well as an outlook on future research.

Chapter 2
Operation and Regulation of Share Repurchases

Share repurchases can be conducted in different ways: fixed price tender offers, Dutch auctions, or open market repurchases (hereafter OMRs). A significant fraction of share repurchases were fixed price tender offers and Dutch auctions before the middle of 1980's. By 1994, OMRs represents over 95% of repurchase activity (Ikenberry/Vermaelen, 1996). From 1998 to 2001, over 68 countries, including the ten largest stock markets, implemented regulations and amendments to enable OMRs. It has since become the most popular corporate payout method (Kim *et al.* and Grullon/ Ikenberry, 2002).

Table 2-1 Number and Value of Share Repurchase Announcement in the USA

Year	Dutch Auctions		Fixed Price Tender Offers		Open Market	
	Cases	Dollars (mills)	Cases	Dollars (mills)	Cases	Dollars (mills)
1980	-	-	1	5	86	1429
1981	-	-	44	1329	95	3013
982	-	-	10	1164	129	3113
1983	-	-	10	1352	53	2278
1984	1	9	67	10519	236	14910
1985	6	1123	36	13352	159	22786
1986	11	2332	20	5492	219	28417
1987	9	1502	42	4764	132	34787
1988	21	7695	32	3826	276	33150
1989	22	5044	49	1939	499	62873
1990	10	1933	41	3463	778	39733
1991	4	739	51	4715	282	16139
1992	7	1638	37	1488	447	32635
1993	5	1291	51	1094	461	35000
1994	10	925	52	2796	824	17036
1995	8	969	40	542	851	81591
1996	22	2774	37	2562	1111	157917
1997	30	5442	35	2552	967	163688
1998	20	2640	13	4364	1537	215012
1999	19	3817	21	1790	1212	137015
Source: Grullon/ Ikenberry (2002) What do we know about stock repurchase?						

Share repurchases are generally announced prior to the share buyback programme to signal the company's private information. To avoid Issuers from manipulating market operations and misleading general investors' judgment as regards publicly available information, the share buyback regulation governs the whole process of executing the share repurchase programme starting from ex ante disclosure, approval, repurchase conditions - timing condition, price condition, and volume condition to the ex post publishing of the financial statement. The regulation is different between countries. In order to properly analyze stock repurchases it is therefore necessary to investigate the regulations applicable. This then allows to make realistic assumption for the development of a model in subsequent chapters.

2.1 Forms of Repurchases

Stock repurchases can be conducted in one of three main forms: by tender offers, private negotiation repurchases, or open market repurchases.

Tender offers can be conducted in two different ways: fixed price tender offers or Dutch auctions. A fixed price tender offer is a broad solicitation by a company or a third party to purchase a substantial percentage of company's shares or units during a limited period of time. The offer is at a fixed price, usually at a premium over the current market price, and is contingent on shareholders tendering a minimum number of their shares or units.

Under the Dutch auction share repurchase method, the firm sets a range of prices for which each shareholder who elects to offer the shares to the company must select a single price at which to tender and has three to four weeks to do so. At the end of the offer period, those offers that exceed either a minimum price or such that a maximum number of shares are bought, are bought by the company. Therefore, all shareholders are paid the same price which is the clearing bid or lowest bid accepted (Gay *et al*, 1996). The company generally reserves the right to repurchase more than the specified amount if it desires to do so. The United States is one of the few countries that allow corporations to make tender offer for their own shares at a premium above market prices.

Private negotiation is the least common repurchasing method and as the name implies, it involves the repurchase of stock from large holders through direct negotiations. In this type of transaction, either the shareholder or the corporation may initiate the transaction negotiations.

In OMRs, a corporation gradually repurchases shares using open market transactions through an established broker. The transaction mechanism of OMRs is the same as that of the general public. Issuers announce a buyback price and amount, then execute the order through the market maker to conduct the repurchase. To prevent the illusory creation of widespread demand and hence the problem of price manipulation, the authorities regulate the relative provisions in terms of trade timing, price, and volume to provide a safe harbour to public investors. For instance, share repurchase within 30 minutes of the market opening and closing times are prohibited to prevent companies from controlling opening and closing prices. Volume is also limited to 15 percent of the average daily trading volume.

Different repurchase approaches have arisen to cope with the diverse intentions of share repurchase in each corporation.

2.2 The Importance of Share Repurchases

Share repurchases have increased both in absolute and relative terms in proportion to the use of cash dividends in returning cash to shareholders (Ikenberry/Grullon; 2000). In the United States, share repurchases were a small percentage of cash dividend payouts in the 1970s. However, there has been a large upward shift in share repurchases since in 1984. Between 1984 and 1988, cash dividends have grown at a rate of 9.3% a year; gross share repurchases have grown at a compound annual growth rate of 13.6% per year. Studies also demonstrate that share repurchases have grown at a higher rate than cash dividends since 1980 (Ikenberry/Vermaelen; 1996); especially between 1995 and 1998, gross share repurchases grew at a compound annual rate of over 26%, compared with below 11% for aggregate cash dividends.

Apart from the significant growth of share repurchases in the United States, share repurchases have also been broadly permitted in other regimes which have undergone liberalization. For example, share repurchases have been implemented in , UK in 1981, Switzerland in 1992, Japan in 1995, Canada in 1998, France in 1998, Germany in 1998, Hong Kong in 1998, Italy in 2000, Taiwan in 2000, and Netherlands in 2001. Based on the world stock market capitalization, the countries permitting OMRs make up over 86 percent of the world stock market capitalization.[1]

[1] As recorded by the World Bank 2000.

Baker *et al.*(1981) and Cook *et al.* (2003), among others, even pointed out that companies have adopted share repurchases as an aggressive investment tool (the 'market timing' motive) rather than as a passive cash distribution method. More academic studies, present in the following chapter, document OMRs' power functions to revolutionize current financial operations. Whatever the viewpoints on the applicative volume, and the worldwide adoption of share repurchases, OMRs has changed current corporate finance operations and share repurchases have at least partially become a corporate investment consideration.

2.3 Regulatory Framework for OMRs

As mentioned above, OMRs are not permitted without certain restrictions. This section aims to examine the repurchase regulations of selected countries in order to analyse both significant repurchase conditions and associated flexibilities which include approval of repurchases, timing or duration repurchase condition, price condition, volume condition, and managerial flexibilities. The sample countries are the United States, Japan, UK, France, Germany, Canada, Italy, Netherlands, Switzerland, Hong Kong, and Taiwan. All the investigated countries and conditions are presented in Table 2-2.

(Insert Table 2-2 about here)

2.3.1 Approval of Repurchases

On the approval process in these sampled countries, there are three different types of approval processe: (i) no required approval (ii) shareholder meeting approval and (iii) board approval. Different approval methods provide different magnitudes of flexibility in the launch of share repurchases. The countries without the need for requirement of approval are the US and Japan. These countries have the most flexibility to execute share repurchases. They can react to market price changes immediately in order to stabilize the market share price. The countries that adopt the board approval method are France, Taiwan, Switzerland, and Canada. These four countries must still report relative repurchase information such as repurchase purpose, repurchase prices, repurchase period, and repurchase volumes to the board. Only when given the approval from the board can these companies execute their share repurchase programmes. The most inflexible approval method relates to seeking share repurchase approval during shareholder meetings. The shareholder meeting is usually held annually. The executive managers generally have to report their next year repurchase programme to the shareholders and seek their approvals. These countries are UK, Netherlands, Italy, Germany, and Hong Kong.

2.3.2 Timing and Duration of Share Repurchases

Countries have different regulations considerations regarding the corporate sector's ability to time the market using OMRs. The United States and Japan focus on the limitations of daily transaction. The other sampled countries focus on the time factor of repurchase duration. Timing condition restricts the periods during which issuers may bid for or purchase its common stock. Duration of repurchases is considered to be a significant indicator of the direction of trading, the strength of demand, and current market value of securities. Such consideration is similar to an investment consideration. The time restriction regulation adopted by the United States and Japan is intended to prevent speculators from manipulating the market price. For instance, repurchase trading is prohibited for the last 30 minutes of a trading day (Grullen, 2000).

The second type of restriction is repurchase duration. Although OMRs are not an obligation, the authority gives a duration constraint to manage the company's right to buy back their outstanding shares. Companies can arbitrarily execute their buyback strategy during the valid time period. Otherwise, they are not allowed to do it .The duration of a share repurchase programme is normally between 12 and 18 months and is dependent on the particular country's rules. Countries which have this time restriction are UK, Netherlands, France, Italy, Germany, Taiwan, Switzerland, Hong Kong, and Canada. Other countries such as France and Hong Kong do not only restrict the duration of repurchase, but they also embody a stricter repurchase timing provision within a smaller time interval.

2.3.3 Price Restrictions

Price conditions govern the price of share repurchase. It prevents issuers from manipulating price through repurchase activity. For example, the repurchase transactions cannot be made at a price that is higher than the most recent closing price. In terms of price restrictions, except for the United States, Switzerland, and Hong Kong, countries such as Germany and Netherlands have regulations that ensure that the repurchase price follows the permitted highest buyback price in the shareholder meeting. Canada and Italy stipulate that the buyback price is no higher than the recent high market price while in France and Taiwan the buyback price cannot exceed the daily highest price. Japan and UK also have similar conditions regarding buyback price. Thus, each country has regulations as regards the buyback price in order to prevent potential market manipulation.

2.3.4 Limits on Repurchase Volume

Repurchase volume conditions limit the amount of securities that issuers may repurchase in a market in a single day or within a valid execution period. Volume conditions are designed to prevent issuers from dominating the market as per its securities through substantial purchasing activity. Similarly, there are various caps on repurchase share volume in all sampled countries. In this regard, Canada has the strictest condition in relation to repurchase volume. It requires that the company cannot repurchase more than 10% of its public float, 5% of its total shares, or 2% in 20 days. France, Italy, Taiwan, and Hong Kong also have repurchase volume constraints to the total shares and daily or monthly transaction volume. Other countries have either repurchase volume constraint to total issued shares or daily transaction, the restriction in the United States on repurchasing no more than 25% of the repurchase volume in a single trading day has been suspended in September 2001. The UK has a volume constraint of no more than 15% of its total shares.

2.3.5 Managerial Flexibility

In contrast to repurchase provisions, flexibilities of OMRs have not been legislated in many formal documents. Although companies in many cases establish repurchase plans[2], which state the intended repurchase amount and maximal price, the duration of execution, companies still have the managerial flexibilities to react to the changes in open market, so as to adjust their repurchase strategy. For instance, issuers may postpone the repurchase until conditions are more favorable (deferral option), decide to repurchase a smaller amount than originally planned (reduction), or even abandon the plan (abandonment option) before the end of the repurchase period. Thus OMRs have a large number of flexibilities and are not a fixed commitment, allowing managers to react to changed market conditions by varying the amounts repurchased as well as ending and re-starting the repurchase activity. To conclude, OMRs are not a firm commitment but have many flexibilities and managers can use these embodied flexibilities to create the maximum profit for their shareholders.

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[2] Repurchase Plan: Programme through which a company buys back its own shares in the open market. Most companies announce their repurchase plan which serves as an indication that the corporation's management believes the company's stock price to be undervalued. Also, it reveals the future repurchase details to the investing public. It is announced before their execution.

2.4 Summary

Stock repurchases have increased in importance considerably over the last years and have become an important tool in corporate finance. While several forms of repurchases exist, the most common form has become open market share repurchase in which the company buys its own shares in the market.

Share repurchases are regulated in various ways, differing across countries. By putting restrictions on the prices companies can pay for their own shares, the amount repurchased and the timing of repurchases the regulators seek to ensure a minimum of investor protection. Despite these regulations, companies and their management are given a large degree of freedom and flexibility in the conduct of share repurchases; therefore any modeling of share repurchases has to adequately include these flexibilities. In the following chapters we will thus argue that using a real options approach is a useful way to address such flexibilities.

Chapter 3
Theories on share repurchases

In this chapter, we will review the current academic literature on share repurchases. We organize the literature by the different functions attributed to share repurchases: share repurchases as tools in financial decision making, as a signaling device, as a device to reduce moral hazard, as an investment decision, and finally we discuss the contingent valuation of share repurchases.

The investigational scope of this review makes references across a single classification as defined in this chapter. Besides, this reference review mainly focuses on the study of OMRs. The other study regarding different share buyback approaches such as representative studies of tender offer and the Dutch auction will also be selectively reviewed in this chapter.

3.1 Share repurchases as tools in financial decision making

Research over the last two decades have found that stock repurchases have contributed a significant proportion to internal financial corporate governance, i.e. as a means of cash distribution to shareholders, capital structure fine-tuning, takeover defense, and tax optimization. Implementation of stock repurchases has an impact on the cash levels of companies as well as their capital structure. Research in this area was initiated by Woods/ Brigham (1966) and other significant studies in this field are further surveyed in the remainder of this section.

3.1.1 Share repurchases as substitutes for dividends

Historically, companies have used dividends as a means of returning cash to their shareholders and hence dividend payments are viewed as a form of shareholder value creation. These dividends may be paid out directly to shareholders in two ways, either through cash dividends or share dividends. The shareholder's wealth is increased from the receipt of dividends from the company. Share buyback reduces the number of outstanding shares and hence increases the earnings per share that could be attributed to existing shareholders, so that shareholder value increases in that perspective. Both approaches are perceived to be equivalent methods of shareholder value creation, i.e. economic equivalents, provided that the share buyback is implemented at attractive price levels.

Woods/ Brigham (1966) concluded that the long-run and short-run situations can have different implications with respect to stock repurchases and dividends. Both types of corporate money distribution policies can be manipulated to reach the maximal value target. Ofer/Thakor (1987) documented that stock repurchases and dividend distributions are employed at different times and the two functions independently. As regards the suitable opportunity of executing stock repurchases, Williams (1988), Brennam/ Thakor (1990), and Grullon/ Michaely (2000) documented that stock repurchase is an intermediate financial policy where a firm confronts external economic changes during the span of a business cycle. Brennam/Thakor (1990) found that dividends are likely to be the choice for the smallest distributions, and that tender offer repurchases will dominate very large cash distributions. Li/McNally (1999) indicated that a firm prefers open market repurchase in times of market turbulence or weak business conditions. Regarding the substitution hypothesis, Grullon/MichNally (2002) revealed that American companies are gradually increasing the payout on stock repurchase programmes. However, the growth of dividend forecasts is negatively correlated with share repurchase activity which verifies the substitution hypothesis.

The Modigliani-Miller Irrelevance Theorem states that dividend payments as well as share repurchases should not affect the value of a company and thus it should be futile to investigate their impact. In the presence of agency costs and asymmetric information as well as inefficient markets this has however not to be true and dividend payments as well as share repurchases are relevant.

3.1.2 Adjusting the capital structure

Research in this field has found that repurchase approaches generally take the form of tender offers and Dutch auction methods when used for buying back larger portions of stock and OMRs are employed for buying back a smaller portion of shares. However, it is possible that corporations repurchase shares from the open market to avoid significantly impacting their financial leverage ratios in any one single repurchase programme. Research in this field can be roughly sorted into two different study approaches: one studies the risk of executing stock repurchases while the other studies the effect of executing stock repurchases on the internal corporate structure. Guffy/Schneider (2004) has also found that companies use share repurchases to adjust its financial leverage benefit via the tax advantage of debt financing.

With the hypothesis that stock repurchase is a form of financial intermediation, Leland/Pyle (1977) indicated that executing share repurchases carries a lower risk to adjust a corporate capital structure. Share repurchases can significantly decrease a firm's total risk or return volatility. Besides, the actual buyback trading activity of a repurchase firm is negatively associated with the volatility change and the CAPM beta change (Hertzel/Jain, 1991; and Kim *et al.*, 2003). Jensen (1986) declared that repurchases use up a firm's free cash flow, and hence, mitigate the agency costs associate with the separation of ownership and control to prevent the agent from engaging in negative net present value. Billingsely *et al.* (1989) have also studied share repurchase behaviour on bank holding companies (BHC). Their studies have pointed out that empirical share repurchase announcements are associated with an increase in the total riskiness of BHC stock.

3.1.3 Share repurchases as a takeover defence

Vermaelen (1984) used quantitative methods to investigate the use of share repurchases as a tool of takeover defence. Managerial incentives which are related to preventing take-over bids and executive stock options are taken into account as the decision parameters when it prices securities. Bagnoli/ Lipman (1989) provided the only analytical study of stock repurchases serving as a takeover defence measure by signaling private information. Their study is similar to the general signaling model, taking the value-maximum as a main objective. However, since the managers' incentives have been defined as defensive uses, their study implies that there are too few takeovers for business operation efficiency. There is a different research approach, which is based on the use of economic and financial theory to establish an analytical model to explain how the OMRs can serve as a takeover defence measure. Depending on supply-demand theory, Bagwell (1991) simply derived an equilibrium model to explain that managers can employ share repurchases as a tool to deter takeover. Sinha (1991) assumed that managers can use debt-financed share repurchase as a takeover defence. The optimal level of share repurchase is the result of a trade-off between the benefit of a reduced probability of takeover and the cost of an increased probability of bankruptcy.

3.1.4 Tax optimization

It is rare that an analytical model can be used to explain the tax savings advantage of executing share repurchases since different countries/states adopt different taxation regulations. Most studies in this field are based on empirical studies. Woods/Brigham (1966) pointed out that employing share repurchases probably can eliminate equity costs than through the use of dividends. Masuli (1980) indicated that the tax shield provides a significant incentive to execute share repurchases. Bagwell/Shoven (1989) revealed that share repurchases save more tax payments than dividend issues. Green/ Hollifield (1999) also found that personal tax savings in corporate decisions can also have the same research result. Considering the economic cycle, McNally (1999) conducted a similar study but failed to provide a clear incremental explanation. This tax shield is contingent on different taxation hypotheses and regulations. Despite this the controversial research conclusion, there is no other references documented that indicate stock repurchases to have an adverse effect on corporate taxation or personal taxation. Grullon/Michaely (2000) find that there is a 26-28 percent tax shield advantage in using stock repurchases rather than through dividend policy.

3.2 Share repurchases as a signalling device

Signalling is a method that corporate managers use to reduce information asymmetry. There are occasions when corporate managers have good news about future company profitability but which is not yet reflected in the prevailing stock price since investors only have access to publicly available information. This results in a stock being priced below its intrinsic value. Consequently, corporate managers would attempt to eliminate this form of pricing discrepancy by publicly announcing the good news to investors with the subsequent upward adjustment of the stock price to reflect this incremental new information. Share repurchases, as such, also represent a form of signalling in that it signals to investors that the company's share price is undervalued. The early signalling literature on share repurchase focuses on repurchase tender offer (Dann,1981; Vermaelen, 1981) and develop models consistent with the signaling of share undervaluation. Miller/Rock (1985) pointed out that both stock repurchases and dividend issues are able to signal these companies' better information to support their outstanding share price. Dann (1981) and Vermaelen (1981) find stock repurchases to be a valid method to salvage the undervalued stock price. Furthermore, Bhattacharya (1979) argued that managers will use stock repurchases to save the undervalued price and forgo a profitable investment.

Recent studies have focused on stock price behaviour surrounding open market share repurchases in different circumstances such as a manager's incentive, market participants' response and repurchase strategy to corporate internal financial devices.

McNally (1999) pioneered a significant research framework to study how stock price behaviour is affected when the repurchase amount is varied, and when risk and a firm's earning form part of the endogenous variables. Isagawa (2000, 2002) adopted a similar framework to study the manager's private benefits and long term stock price performance. In his previous study (Isagawa, 2000), he focused on managers' private benefits when real investment opportunities are unprofitable in terms of firm value without the restriction that the announcement are commitments. Later, Isagawa (2002) demonstrated why a firm still executes its repurchase programme even when the stock price is raised. Depending on the asymmetry information assumption and different market participants' responses, Isagawa (2002) believes that a firm may achieve a long-term profit from the execution of its share repurchase plan.

Grullon (2004), Maxwell/ Stephens (2003) overlook the OMRs signaling effect to the stock price and bond price changes. In explaining the returns, the bond returns are more negative and stock returns more positive for large share repurchase programmes. Howell/Payne (2004) studied the rational sequence of announcement and repurchase action. They find that repurchases in the context of a previous stock (i.e. based acquisition) have a less positive market reaction than do otherwise comparable repurchases with no acquisition.

3.3 Share repurchases as a device to reduce moral hazard

The separation of ownership and management causes agency problems. For example, managers may allocate excessive company risk for over-investing or over-investing in unprofitable projects. Share repurchases can reduce free cash flow to reduce the moral hazard of an investment (Danies/Danies, 1993), Lang/Litzerberger (1989) and Lie (2000). They maintain companies that execute share repurchases have a higher Tobin's Q[3]. Repurchase announcements reduce management's ability to divert capital to uses that are not in the best interest of shareholders. Share buybacks signal that management has a focus on shareholder value creation, because it is returning cash to shareholders and sends a signal that management is not engaged in non-value enhancing activities.

Jolls (1998) studied the incentives of the agents, the issuers, to explain the puzzles concerning a firm's payout behaviour, and also studied the effects of executive compensation package on managerial incentives. The stock option-based restricted stock is significantly more likely to be a form of compensation to top executives. Such restricted stock compensation is not diluted by dividend payments in his

Tobin's Q: represents the value of a company given by financial markets divided by the value of a company's assets. If the market value reflected solely the recorded assets of a company, Tobin's Q would be 1.0. High Tobin's Q values encourage companies to invest more in capital because they are "worth" more than the price they paid for them.

sampled firms. Weisbenner (2000) studied the effect of general share repurchase from the open market by different parties. He concluded that the larger the option held by top executives, the more apt the firm is to retain more earnings and curtail cash distribution.

3.4 Share repurchases as investment decisions

The most frequently cited reason for initiating an open market share repurchase programme is that a firm considers its stock to be undervalued and is therefore a good investment (Cook *et al.*, 2003), i.e., by buying their own companies' shares, they can expect a high return. Information asymmetries continue to exist between corporate managers and their outside investors. As such, the former generally have better information and are therefore able to make a good investment simply by buying back their undervalued shares. The existence of such undervalued shares implies that the market is not efficient, either in the strong or semi-strong form. In an efficient market the timing of repurchases to make profitable investments would not be possible. Research in this field can be categorized into four main research approaches. Firstly, most of these studies, such as Masulis (1980), Vermaelen (1981), Dann *et al.* (1991), Hertzel/Jain (1991), Dharan/Ikenberry (1995), Ikenberry *et al.* (1995), Ikenberry/Vermaelen (1996), Lie/McConnell (1998), and Cook *et al.* (2003) addressed the share price changes surrounding the share buyback announcement. The consensus is that there is a negative return surrounding stock repurchases before the announcement of the share buyback; then there follows a small positive short-term return. The share repurchases have a much more significant impact as regards the long run performance. The literature concluded that the above research results are a consequence of the initial announcement and share repurchase reaction which indeed provides some support for the mispricing story. Initially the market is slow to react to this information due to market inefficiencies. In the long run, however, the market fully adjusts.

Secondly, the stock repurchases offer a form of flexible manipulation in a future environment of uncertainty. It is not a commitment policy, and firms are authorized to decide when and how much share buyback is to be completed in an environment of uncertain market change. A study conducted by Kirch/Barnic (1998) concluded that the non-fulfilling firms are more likely to incur a poorer stock return than fulfilling firms. This result bears an inverse effect to the principle of the flexibility of share repurchases.

Thirdly, recent repurchase studies have started to discuss the applications and the influence of repurchased shares on stock options. Jolls (1988) studied the effects of the issue of different types of stock options on the firms' future earning per share (hereafter EPS) change and managers' incentives. He found that firms which rely heavily on stock-option-based compensation are significantly more likely to repurchase their stock than firms which rely less heavily on stock options to compensate their top executives. Further, Jolls (1988) found no such relationship between repurchases and restricted stock, an alternative form of stock-based compensation that, unlike stock options, is not diluted by dividend payments. These findings have implications for the study of other puzzles concerning a firm's payout behaviour as well as the study of the effects of executive compensation package on managerial incentives. Weisbenner (2000) found that an analysis of panel data for a sample of large firms suggested that firms conduct ongoing repurchases of shares over the life of an option which undo much of the dilution to EPS that results from past stock option grants. Option grants in general are associated with increased share repurchases and increased total payouts. However, the larger the executives' holding of stock options, the more apt the firm is to retain more earnings and curtail cash distributions. Weston/Siu (2002) believed that stock prices represent the discounted value of expected future net cash flows providing the ability to make large gains.

Hence it has the ability to benefit from the capitalized value of expected favorable future earning growth that makes stock options valuable.

Finally, the executive timing issue has been emphasized during the past few years. Stephens/Weisbach (1998) examined quarterly repurchase volume in the United States and found that the repurchase is negatively correlated to prior returns and positively related to both expected and unexpected future cashflows. Ikenberry *et al.* (2000) documented that a set of Canadian firms increased their repurchase activities in the months following a price drop. Cook *et al.* (2003) documented that the timing of OMRs execution in the United States is related to the bid-ask spread, liquidity, and firm-specific information. Ginglinger/Hamon (2003) studied the share price performance of French companies following share repurchases. Fairchild and Zhang 2006) develop a model of repurchase timing in which investors are slow to react to the announcement and conduct of share repurchases. This market inefficiency allows managers to time the market and depending on conditions they are finishing their repurchase activities either over short or long time periods.

Their results show that the managerial timing ability is far from uniform. On average, firms on share repurchase disclosure do not succeed in repurchasing shares at a lower price than that by other investors. Put simply, the issuers ignored timing flexibility. Therefore, most of them failed to capitalize on this valued flexibility.

3.5 Contingent valuation of share repurchases

OMRs are conducted in an uncertain public market, their contingent value demonstrate stochastic behaviour. A deterministic pricing method fails to catch such risk premium investment. Ikenberry/Vermalen (1996) identified OMRs as an exchange investment and pioneered a contingent value study on price as a fundamental basis for further study. The contingent value of stock repurchases includes the market price increase that results from the substantial outstanding share reduction and the value of outstanding shares exchanged to internal corporate holdings. The appraisal technique of exchange value is based on Margrabe's study (1978) "The Value of an Option to Exchange one for another", which assumes that the underlying transaction is in compliance with a type of European Option.

3.6 Summary

The literature documents that share repurchase programmes in corporate internal governance can be used as a means of dividend substitution, pro-active takeover defence, tax saving, and corporate structure fine-tuning. However, the application of stock repurchases to external share return has had controversial study results. Generally speaking, post-announcement of a share buyback, a small positive short term return results which is then overshadowed by a much more significant long run performance.

The interest of shareholders would imply that the manager executes the stock repurchase such that the profits are maximized. In the presence of agency problems, however, the manager would tend to maximize his own profits, which might result in actions which are contrary to those of the shareholders. To form the optimal model, he is aware of the need to analyse the interrelationships between its functions. The relationship between share repurchases and dividend distribution is a time lag function, and they are independent of each other within the same time-frame. Therefore, managers can simply consider how to execute this financial policy as a single objective when managers plan to distribute money to shareholders.

It is hard to use an analytical model to predict how much tax can be saved by executing stock repurchases, thus an empirical study is recommended to evaluate the amount of tax savings. It is possible to exclude the issue of tax savings alone when deriving a maximum contingent value for a valuation model. Besides, as regards the impact of stock repurchases on the external stock return, the majority of references documented that stock repurchases only have a short term benefit. This argument implies that stock repurchases can only be adopted to yield a contingent value and therefore managers can adopt this policy to precede a takeover defense at a lower price. Finally, the functions of pro-active takeover defense and corporate structure fine-tuning can be ruled out as long as we can optimise this programme.

Given the variety of possible motives for share repurchases, it is in all cases important for the managers to conduct the share repurchase in the best way possible, e.g. through an optimal timing and repurchase strategy.

Ikenberry/Vermaelen (1996) use option-based theory to pioneer the contingent evaluation of the OMRs. They adopted a simple exchange option with assumptions of a fixed transaction day, and a given strike price quotation, so that their model is weak in explaining the inherent stock repurchases' flexibilities; OMRs is a non-commitment American-type call option with repurchase timing/duration, repurchase price, and repurchase volume conditions/ flexibilities. To reach an optimal performance of executing OMRs, managers have to treat executing OMRs as a form of investment behaviour. Managers have to depend on the best timing and the best trigger price to create benefits for their long-term shareholders. Such derivative financial behaviour which derives their values from the underlying(s) in an uncertain risky market has become an important focus in modern financial markets. And this subject has also provided challenging research problems. Aimed at closing this research gap, this study will investigate an optimal share repurchase trigger timing model by option-based pricing techniques.

Chapter 4
The theory of real options

The real-options approach applies financial options theory to real investments, such as manufacturing plants, product line extensions, and research and development. A financial option gives the owner the right, but not the obligation, to buy or sell a security at a given price. Analogously, companies that make strategic investments have the right, but not the obligation, to exploit these opportunities in the future. For instance, a company may postpone a share repurchase until conditions are more favourable (deferral option), decide to repurchase a smaller amount than originally planned (option to contract), or even abandon the plan (abandonment option) altogether. Those discretions are valuable and they often govern whether a project is survivable and ultimately deliverable.

Unfortunately, the traditional capital evaluation model, the net present value (NPV) rule, leads to a suboptimal investment decision when part of the investment costs are already sunk and managers have some discretions about timing flexibility. An option-based pricing technique is therefore established to adapt to such flexibility in order to conduct future actions in response to altered future market conditions and to enhance an investment opportunity. To evaluate investment flexibility, developments of financial and real options pricing techniques have slowly matured to incorporate complex financial projects and real asset investments. The essence of the analytic techniques of real options, commonly known as real option analysis (hereafter ROA), is to combine the strategic intuition and analytical rigor into an advanced pricing technique.

4.1 Real vs. financial options

In general, the financial option pricing technique has been employed to price the closed form European type derivatives such as the European option. A financial option gives its owner the right but not an obligation to purchase or sell a security at a given price. Real option analysis extends financial option theory, of which the most famous model is the Black-Scholes model, with strategic applications to options on real or non-financial assets. In other words, ROA is derived from financial theory with considerations of the investment strategies to be a new financial pricing technique. The commonality can be supported from the two classic option-based studies conducted by Merton (1973), and Dixit/Pindyck (1994). Both the authors assumed the option value to be contingent on one or more underlying asset(s), and its instantaneous change is a Brown-Wiener motion. Depending on Ito's Lemma and the mathematical optimal theory, the diffusion of option value can be obtained from both financial option pricing techniques and real option analysis.

Despite ROA and financial options being developed to evaluate the flexibilities of an investment, they, however, have differences in their assumptions and applications. The Black-Scholes model is well-known to incorporate five parameters to appraise the option value, being the rate of return, variance, time to maturity, stock price, and the strike price. The measurement of variance in financial options is usually derived from the easily observed historical prices of the underlying assets. But there are almost by definition no historical numbers to be used when managers evaluate a real estate investment or an innovative project. Besides, the future investment cost is hard to obtain for a real investment. For example, the buyback cost of executing OMRs is decided by the spot price at that time of purchase which is in turn decided by market participants. There is no market quotation of a strike price at the expiry date or the date that issuers intend to buy, i.e., the project is done at a cost equal to the investment required. Finally, there is a distinguish between financial option and real options. With a financial option, the more time we have before we commit to buying the underlying asset, the more valuable the option. This makes sense because the stock has more time to increase in value, and if it does not, we need not exercise, so financial options with longer expiration periods have more value than those with shorter lives (all other things being equal). This logic does not extend to the real world, however. The option to delay a product launch will not necessarily add value to a project because the investor may end up paying a discount penalty and could even end up missing the market altogether. The relationship between time and value is much less consistent with real options than it is with financial options (van Putten/MacMillan; 2004).

4.2 The basics of real options analysis

The development of a continuous-time of ROA pricing technique has a consolidated process of model setup and a solution process. The process of model setup incorporates the stochastic financial theory, expectations theory, and the mathematically optimal theory. The solution process employs complicated engineering mathematics to solve the optimal decision making model (system).

In the first stage of model setup, one needs to make assumptions on the option value, defined as $F(S,t)$, which is contingent on the underlying asset, S. It is assumed that the underlying asset follows a simple Brown-Wiener process in continuous time:

$$d(\ln S) \approx \frac{dS}{S} = \alpha_S dt + \sigma_S dz \qquad (4\text{-}1)$$

where

α_S : drift of underlying asset

σ_S : variance of asset

dz_S is a standard Brown-Wiener process.

A contingent value is considered as a function that is twice differentiated with respect to S and once with respect to t, and this is then denoted as $F(S,t)$. Eq (4-1) is not differentiable, so we need to work with function of dynamic programming to let the function of contingent value exist as an optimal function. The expansion function of this contingent value is:

$$dF = \frac{\partial F}{\partial S} dS + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2 + \frac{\partial F}{\partial t} dt \qquad (4\text{-}2)$$

Eq.(4-2) has an optimal stopping problem in continuous time because an investment opportunity, $F(S,t)$, yields no cash flows up to the time of exercise (T) that the investment is undertaken, and the only return from holding it is its capital appreciation. Hence in the continuous region, the above exercise strategy can be written as Eq. (4-3)[4]

$$\rho F dt = E(dF) \qquad (4\text{-}3)$$

where:

ρ: expected rate of capital appreciation

Eq. (4-3) is the Bellman's Principle of Optimality[5] which indicates that over a time interval dt, the total expected return on the investment opportunity, $\rho F dt$, is equal to its expected rate of capital appreciation. Substituting Eq. (4-1) into Eq. (4-2), we then cam have

[4] For facilitating the following derivation, the sub-notation is ignored

[5] Bellman's Principle of Optimality: An optimal policy has the property that, whatever the initial action, the remaining choices constitute an optimal policy with respect to the sub-problem starting at the stated results from the initial actions

$$\frac{1}{2}\sigma_s^2 S^2 F_{SS} + \alpha S F_S - \rho F = -F_t \qquad (4\text{-}4)$$

where

F_S : is the first differential of F by underlying price

F_{SS} : is the second differential of F by underlying price

F_t : is the first differential of F by time

In a real investment consideration, the instantaneous change of the option value in time factor could be vanishing in two scenarios. One is that the factor has only a slight influence on the option value. People can simply ignore its existence. The other scenario is that a real investment can last for longer project life. The option has infinite life, i.e., the project does not expire and thus time is irrelevant. In this case, Eq. (4-4) can be derived as

$$\frac{1}{2}\sigma_s^2 S^2 F_{SS} + \alpha_S S F_S - \rho F = 0 \qquad (4\text{-}5)$$

Eq. (4-5) is the Black-Scholes differential equation without considering the influence of the time factor. Eq. (4-5) has many solutions, corresponding to all the different derivations that can be defined with S as the underlying variable. The particular derivative obtained when the equation is solved depends on the boundary conditions that are used. These specify the values of the derivative at the boundaries of possible value of S. In the case of an investment with an investment cost, C, the key boundary condition that we can observe is the payoff from exercising the option and this is given by

$$F(\mathrm{S},t) = S - C \tag{4-6}$$

and

$$F_S(S,t) = 1 \tag{4-7}$$

and

$$\lim_{S \to 0} F(S,t) = 0 \tag{4-8}$$

As in American option pricing problems of this sort, there will be a trigger value, S^*, where the investor will exercise the option to trigger his/her investment to create the maximal profit for their shareholders.

The first boundary condition is commonly termed the "value-matching" condition. The instantaneous profit from exercising option has to equal the value of option to wait. In that the issuers will receive a net payoff $S^{*}-C$. The second boundary condition is known as the "high-contact" or "smooth-pasting" condition. Essentially, this condition ensures that the trigger is optimal. Final boundary conditions are imposed: $\lim_{S\to 0} F(S,t)=0$. This is the result of the fact that if the value of investment goes to zero, the investment will never be exercised and will therefore become worthless. So far, the dynamic optimal pricing model, a homogeneous second order differential model, has been set up. The complete model derivation is presented in the Appendix A.

Eq.(4-5) is an Euler equation, which has a general solution

$$F = A_1 S^{\beta_1} + A_2 S^{\beta_2} \tag{4-9}$$

Eq.(4-7) restrains that the A_2 has to be zero (the $A_2 S^{\beta_2}$ is not satisfied by the boundary conditions of $\lim_{S\to 0} F(S,t)=0$) so that leaves the general solution of Eq. (4-9) as

$$F = A_1 S^{\beta_1} \tag{4-10}$$

Substituting Eq. (4-9) into Eq. (4-4), the value of β can be obtained

$$\beta_1 = \frac{\left(-\alpha_S + \frac{1}{2}\sigma_S^2\right) + \sqrt{\left(\alpha_S - \frac{1}{2}\sigma_S^2\right)^2 + 2\rho\sigma_S^2}}{\sigma_S^2} \tag{4-11}$$

So far, the unbounded optimal contingent value differential equation has been found.

Next, we can rely on the general solution of Eq. (4-10) with the investor's expectations revenue function and the high contact conditions to solve the optimal trigger point of OMRs.

Firstly, by differentiating Eq. (4-11) by S and then letting the new differential equation to be equal to Eq. (4-7), we can solve the equation as:

$$F_S = A\beta_1 \left(S^*\right)^{\beta_1 - 1} = 1$$

or

we can write $A = \dfrac{1}{\beta_1 \left(S^*\right)^{\beta - 1}} = \dfrac{\left(S^*\right)^{1-\beta_1}}{\beta_1}$ (4-12)

Substituting Eq. (4-12) into Eq. (4-10) and letting it be equal to Eq. (4-6), we can solve the optimal contingent value evaluation model with imposed boundary conditions.

$$\frac{\left(S^*\right)^{1-\beta_1}}{\beta_1} \times S^* = S^* - C$$

$$S^* = \frac{\beta_1}{\beta_1 - 1} C \tag{4-13}$$

$$F\left(S^*,t\right) = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{(\beta_1)^{\beta_1} C^{\beta_1 - 1}} \times \left(\frac{\beta_1}{\beta_1 - 1}\right)^{\beta_1} C^{\beta_1} = \frac{C}{\beta_1 - 1} \tag{4-14}$$

and the coefficient of the general solution is given by:

$$A_1 = \frac{S^* - C}{\left(S^*\right)^{\beta_1}} = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{(\beta_1)^{\beta_1} C^{\beta_1 - 1}} \tag{4-15}$$

Eq. (4-13) to Eq. (4-15) give the optimal investment rule, when $S^* \geq C$, the optimal contingent investment value, $F\left(S^*,t\right)$. Furthermore, Eq. (4-11), Eq (4-13), and Eq. (4-15) also illustrate several financial theories. Eq. (4-11) demonstrates that β_1 is bigger or equal to 1, thus the value of $\frac{\beta_1}{\beta_1 - 1}$ is bigger or equal to 1 and $F\left(S^*, t\right) > C$.

This result proves that the simple net present value (NPV) rule is incorrect. The uncertainty and irreversibility derive there is a wedge between the trigger value/ critical value, S^* and investment cost, C.

4.3 Applications of real options analysis

This section will first introduce the issues of real options that issuers will have when they conduct OMRs programme. A further competitive option-based pricing theory will also be discussed as well. Finally, the sequential occurring option model will also be analysed and studied thoroughly.

4.3.1 Investment abandonment and deferral

The conventional capital budgeting approach considers two alternatives: invest now or decline the investment. Usually this approach does not consider possibilities of abandoning a project before completion nor opportunities to defer or postpone an investment. The possibility to abandon a project, if it later turns out to be less attractive, constitutes a valuable option. Similarity, the opportunity to defer an investment decision to times when some of the uncertainties might be eliminated creates a valuable option (Brealey/Myers, 1996).

The investment abandonment and deferral perspectives arguably have two applications in strategic decision analyses. The abandonment option is an option to close out an investment prior to the fulfillment of the original conditions for termination. Formally it is an American put (Copeland/Antikarov; 2000). The abandonment option perspective applies particularly well to retractable investments. The abandonment option is typically constructed as sequential or staged investment paths, so the firm has the opportunity to abandon the project at different points in time during the development period. Researches in this field are mostly in research and design stages. McGrath (1997) developed a real options logic to analyse technology investment decisions. He (1997) provides a thorough discussion of how various sources of uncertainty, e.g. technological risk as well as input cost variability impact investment decisions. Childs/Triantis (1999) examined dynamic research and design investment policies and the valuation of research and design programmes in a real option framework. Schwartz (2004) extended the abandonment valuation method which considers research and design projects and patents via the simulation method.

The deferral option is an American call option found in most projects where one has the right to delay the start of a project. An initial development investment might lead to several business opportunities, each representing different potential irreversible resource commitments. The deferral option perspective is particularly suited to the analysis of irreversible investment commitments. Its exercise price is the money invested in getting the project started. Irreversible investment decisions refer to the subsequent resource commitments on real and intangible assets, e.g. production plants. Researchers in this field include Reinganum (1981), Fudenberg/Tirole (1985), and Riordan (1992). One common feature of this literature is the competition between participants. Two effects are typically present: preemption incentives, which lead to early adoption and information spillovers, which lead to late adoption. Besides, Farrell/Galini (1988) show that a monopolist might want to license its technology to a competitor as a means of commitment not to increase future price and this helps solve a hold-up problem. Cabral/Dezso (2006) incorporated game theory into option to defer to study the optimal competitive strategy amongst different market participants.

4.3.2 Sequential Investment

In many situations, decisions are made sequentially and in a particular order. Such an investment is called a sequential investment. The above deferral option and abandonment option exist in a sequential type investment. The key characteristic of a sequential investment is the ability to temporarily or permanently stop investing if the value of the completed project falls, or if the expected cost of completing the investment rises (If one had no choice but to complete the project once it had been started, investing would once again involve only a single decision.) This probability of stopping midstream makes these investments analogous to compound options; each stage completed (or dollar invested) gives the firm an option to complete the next stage (or invest the next dollar). The investment problem boils down to finding a contingent plan for making these sequential (and irreversible) expenditures.

The compound option has two basic definitions, namely simultaneous compound options and sequential compound options. The key feature of "simultaneous" compound options is that the underlying option and the option on it are both simultaneously available - both options are alive during the same interval of time, the call contingent on the equity is a simultaneous compound option.

In a sequential compound option, on the other hand, one option is created only when the first option has been exercised. Any type of phased investment shall fit this category. For example, most factories can be constructed in several phases: a scheme design phase, a detailed engineering phase, and finally a construction phase. Product development programmes are another example. Usually they have a phase of development, a phase of market test, and then a phase of full-scale product launch.

Such compound options can be intuitively understood since real world investment decisions are generally made sequentially over time. Management must consider the possibility of subsequent decisions like suspending a project when an initial investment decision is made. A sequential option pricing technique aims to evaluate such a real investment. In a sense, the first option chronologically is the right to buy the second option.

Most of the existing references, regarding compounds options, belongs European type compound options study such as McDonald/Siegel (1985) created a contingent value price model for a firm which it allowed temporary or costless terminating a project and restarting at later stage. Sing (2002) studied "time to build options in construction processes". The author analyzed manager's prospects on the value of optimal trigger to construct a building. Even the studies of Majd/Pindyck (1985) and

McDonald/Siegel (1985) are American type call option studies, they lack an empirical verification. The references still stay in the stage of theoretical development. However, developing a manager's prospections to a real investment then formulise the prospections and the flexibilities of investment as a solvable ROA evaluation model still have a long way to go.

4.4 Development and Assumptions of Employing ROA to the optimal trigger timing of OMRs

The aim of this section is to provide a preparation to subsequent model development. Thereby, this section will clearly introduce the options when issuers conduct OMRs. Further, the necessary assumptions of building up the optimal pricing model to OMRs will also be presented. This section aims to provide a preliminary to the following model derivation.

4.4.1 Relationship between ROA and OMRs- optimal trigger timing

Notwithstanding that we have introduced that OMRs has a very similar characteristic to investment projects and are subject to similar flexibilities, to link the options of executing OMRs with ROA, people need to know in practice how this works in a repurchase strategy.

Operating OMRs is the same as general investors trading shares in the open market. The deferral option occurs at the beginning day of repurchasing if issuers do not execute their repurchase rights immediately. Issuers initially start their first time repurchase activity and can proceed to complete this activity. The issuers could also postpone their repurchase activity and wait for the market response before deciding their next repurchase activity or they can wait for the next repurchase when conditions are more favourable. Meanwhile, the deferral option appears again until issuers see the next trigger. Such a repurchase strategy is repeated continuously until the expiry of the authorized repurchase programme. To sum up, when the total buyback volume is less than the announced buyback volume, the option to contract exists. Or we can say that such issuers can abandon their repurchase right before the expiry. Furthermore, as regards the differences between the option to deferral and the option to abandon, these are difficult to distinguish.

The value of a deferral option arises from the possibility of better investment conditions in the future, such that waiting with the investment decision is beneficial.

OMRs is a non-commitment investment strategy; the contingent value will vanish as long as the investment opportunity is gone. Furthermore, to assume that the future stock price has a foreseeable pattern is unrealistic. Therefore, executing OMRs incorporates the option to defer repurchase activities, however the option to defer has no intrinsic value once the trigging timing is expired. Moreover, the flexibilities of OMRs includes an authorized right to buyback the outstanding shares. As long as issuers give up the right, option to abandon, then the authorized rights will vanish and the derived contingent value from an ex ante authorized option becomes worthless.

There is no study incorporating ROA into the appraisal of the optimal trigger timing of OMRs up-to-date. We turn to this study to focus on the optimal trigger timing since it is the most concerning question to an issuer whether to decide to launch OMRs. ROA has an advantage over the traditional financial option pricing technique in that it is able to provide the result of optimal trigger timing (as illustrated in section 4.2). The most important questions to issuers are (i) how much the issuer needs to pay for buying back outstanding shares and (ii) how many shares they need to repurchase in order to reach the maximum profit under the price condition and volume condition. Thus, the following chapter aims to use OMRs to create a sequential optimal trigger timing model.

The optimal trigger point is defined as when the diffusion of the contingent value of share repurchases are tangent at the point between an issuer's expected return. The contingent revenue that issuers can have is the price difference/asymmetry payoff between a company's fundamental value and market price. The total contingent value is decided by how much the price difference can be saved multiplied by the repurchase volume..

4.4.2 Model Assumptions

Prior to the formal derivation of optimal trigger timing to OMRs, it is important to setup the assumptions of model development and our research aims. Furthermore, several assumptions with respect to modeling requirements need to be declared, especially, the type of contingent value, underlying asset(s), transaction characteristics and trading volume consideration.

Firstly, OMRs are not a firm commitment and any pre-announced plan can be abandoned before the predicted expiry date. Therefore, it has been asserted that

OMRs is a type of American option because the share repurchase can be ceased before the expiry.

As regards the strike price issue, despite an issuer knowing the fundamental value, he/she still relies on the current market price and market response in order to decide its strike price instead of taking self quotations. In addition, share repurchases are not similar to other financial derivatives transaction that requires documentary quotations. Therefore, there is no indicative strike price in a repurchase plan or in an executive period. All issuer's quotations depend on uncertain market price changes and the responses from other investors.

Since the option value of OMRs is contingent on the asymmetry payoff between fundamental and market share price, naturally it can be formed as an exchange option. Issuers always seek the optimal trigger timing to repurchase their outstanding shares to create the maximum profit to their long-term shareholders. Besides, both underlying assets are interactive in the relationship of covariance.

The importance of the trading volume with respect to influencing the share price transaction has been documented (Epps, 1977; Tse/Devos, 2004; Fairchild and Zhang, 2005). Most contingent value references ignored this endogenous variable to simplify their studies. To link this study closely to reality, it is first assumed that the optimal repurchase volume will be given to start up our fundamental model setup. Then this fundamental model will be further extended to derive a more advanced model, where the trading volume is also one of the variables.

Chapter 5
A real options model of share repurchase timing

Although the flexibilities of executing OMRs include the option to deferral and the option to abandon, our study aims to provide the investment information for the best investment timing as well as investment volume since these are of utmost concern for investors. By doing this, issuers/ users of our created models need only focus on the optimal trigger timing, so that they are able to create the maximum profit for their long-term shareholders.

In section 5.1, we introduce a one-time repurchase model. The one-time optimal repurchase timing model under given repurchase volume model and one-time optimal repurchase timing model with optimal given repurchase volume are provided. In section 5.2, we delve into the extensive sequential repurchase price model under given repurchase volume model. Section 5.3 will introduce the hypothesis for further empirical investigation and the process of employing the developed models for the practical application. Finally, section 5.4 will address the limitations of our model development and application. The contents of this chapter include assumptions of model development, model setup, solution process, and how to implement the creation of the final model so as to compare our findings against existing literature.

5.1 Single share repurchase

5.1.1 Assumptions of model development

The value of OMRs options is contingent on the asymmetry payoff between fundamental value (V) and market share price (P). Let us consider a firm with a fixed number of N outstanding shares whose net assets have a fundamental value of M such that the fundamental value of a single share is $V = \frac{M}{N}$. It is assumed that this fundamental value, follows a simple Brown-Wiener process in continuous time:

$$d(\ln V) \approx \frac{dV}{V} = \mu_V dt + \sigma_V dz_V \tag{5-1}$$

where dz_A is a standard Brown-Wiener process.

The market price can deviate from this net asset value due to the presence of noise traders whose relative net demand $\frac{S}{N}$ is purely stochastic:

$$\frac{S}{N} = \sigma_S dz_S \tag{5-2}$$

where dz_S is a standard Brown-Wiener process, independent of dz_A. With a market liquidity of λ the market price P follows:

$$\begin{aligned} d(\ln P) &\approx \frac{dP}{P} = \frac{dV}{V} + \lambda\left(\frac{S}{N}\right) \\ &= \mu_A dt + \sigma_A dz_A + \lambda \sigma_S dz_S \\ &= \mu_A dt + \sigma_P dz_P \end{aligned} \tag{5-3}$$

where $\sigma_P^2 = \sigma_V^2 + \lambda^2 \sigma_S^2$ and dz_P is another Brown-Wiener process.

5.1. 2 Derivation of model

1. Given the repurchase volume

Recall that in this model the motivation for launching OMRs is trying exploit the undervaluation of the company and derive the optimal timing for such an action. Therefore, the contingent value of executing OMRs is the price difference between the fundamental value and market share price. Mathematically, the returns from executing OMRs can be represented as $\ln\frac{V}{P}$. Hereafter, this is denoted as $g=\ln\frac{V}{P}$, where g is a real function representing the joint underlying assets V and P. Thus, by totally differentiating g, we can get Eq. (5-4).

$$dg=\sum_i \frac{\partial g}{\partial x}dx_i = V^{-1}dV - P^{-1}dP = \lambda\sigma_s dz \tag{5-4}$$

An OMRs investment opportunity is contingent on the variable g, denoted as $F(g)$, and this yields no cash flows up till the time of exercise when the investment is undertaken. Otherwise, the only return from holding it is its capital appreciation. Hence in the continuous region, the above exercise strategy can be written as Eq. (5-5).

$$\rho F dT = \varepsilon\left(dF\right)^{6} \qquad (5\text{-}5)$$

ρ : expected rate of capital appreciation

Eq. (5-5) is the Bellman's Principle of Optimality which indicates that over a time interval dt, the total expected return on the investment opportunity, ρFdt , is equal to its expected rate of capital appreciation. Then dF is expanded by ordinary derivation, and the optimal contingent value function can be presented as follows:

$$\frac{1}{2}\lambda^2\sigma_s^2F_{gg} - \rho F = 0 \qquad (5\text{-}6)$$

where

F_{gg} : is the second differential of F

Eq. (5-6) is a stochastic ordinary differential equation without considering the influence of time factor. Eq. (5-6), as with Eq. (4-5), also has many solutions that correspond to all the different derivations that can be defined with g as the underlying variable. The particular derivative obtained when the equation is solved depends on the boundary conditions that are used. These specify the values of the

[6] We disregard the sub-notations for simplicity of notation

derivative at the boundaries of possible values of g. In the case of share repurchases take place in one time period, the key boundary condition that we can observe is the payoff from exercising the option at time T. The expected profits from the repurchase of a given amount, Q, of the outstanding number of shares with a liquidity λ of the market are given by

$$F(g) = F(V,P) = N\int_{T}^{T+1}(\ln V_T - \ln P_T)d\eta \tag{5-7}$$

Refining Eq. (5-7), this can be written as

$$F(g) = F(V,P) = QN\left(\ln V - \ln P - \frac{1}{2}\lambda Q\right) \text{or } QN\left(g - \frac{1}{2}\lambda Q\right) \tag{5-8}$$

As in American option pricing problems of this sort, there will be a trigger value, g^*, that the issuers will exercise the option to repurchase their outstanding shares to create the maximal profit for their shareholders.

Upon exercise, issuers will receive a net payoff $QN\left(g^* - \frac{1}{2}\lambda Q\right)$. As the option has not been exercised, its value is positive at any point of time before its exercise. We can immediately derive that and deduce that $g^* \geq \frac{1}{2}\lambda Q \geq 0$. Eq. (5-9), high contact condition, ensures that the trigger is optimal.

$$F_g(g) = F_g(V, P) = QN \tag{5-9}$$

Final boundary conditions are imposed:

$$\lim_{g \to -\infty} F(g) = 0^{7} \tag{5-10}$$

This is the result of the fact that if the market share price goes to an infinite value or the fundamental value ever reaches zero, then the repurchase programme will never be exercised and it will become worthless. A real options model of one-time share repurchase timing, starting from Eq. (5-6) to Eq. (5-10) has been constructed.

5.1.3 Solutions of Process and Results

Eq. (5-6) is a standard ordinary differential equation which has a general solution of

$$F(g) = A_1 e^{\beta_1 g} + A_2 e^{\beta_2 g}. \tag{5-11}$$

---

[7] $\lim_{g \to -\infty} F(g) = 0$ has two scenarios which are $\lim_{P \to \infty} F(V, P) = 0$ or $\lim_{V \to 0} F(V, P) = 0$.

The general solution, Eq. (5-11), of the optimal one-time repurchase model is different from the general solution of Eq. (4-5), Eq. (4-9). Eq. (4-5) is a homogeneous ordinary equation with variable coefficients; it is a type of Euler equation. Hence, the general solution is a polynomial function. However, Eq. (5-6) is a standard ordinary equation with the characteristics of non-repeat to zero solution, so that its general solution is a type of exponential function (Giordano/ Weir; 1991).

The contingent value $F(g)$ also has to satisfy the boundary condition (5-10). Therefore, the second part of general solution cannot be satisfied. The remaining general solution of Eq. (5-6) remains the first part of Eq. (5-11) eventually. We write it as follows:

$$F = A_1 e^{\beta_1 g} \tag{5-12}$$

Then, by an ordinary calculus, to solve the optimal one-time repurchase model, including diffusion equation Eq. (5-6), boundary equations Eq. (5-7) and Eq. (5-9) , and the high contact equation, the following results can be obtained:

$$\beta = \frac{\sqrt{2\rho}}{\lambda \sigma_s} \quad \text{(the negative item is ruled out)} \tag{5-13}$$

$$g^* = \frac{1}{\beta} + \frac{1}{2}\lambda Q \tag{5-14}$$

$$F\left(g^*\right) = \frac{QN}{\beta} \tag{5-15}$$

and the coefficient of the general solution is

$$A_1 = \frac{QN}{\beta} e^{-\beta\left(g^*\right)} = \frac{QN}{\beta} e^{-\frac{1}{2}\lambda\beta Q - 1} \tag{5-16}$$

Substituting Eq. (5-14), Eq. (5-16) into Eq. (5-12), the value of the repurchase option for a fixed repurchase amount Q is given by

$$F(g) = \frac{QN}{\beta} e^{\beta\left(g - \frac{1}{2}\lambda Q\right) - 1} \tag{5-17}$$

Eq. (5-14) includes two components: $\frac{1}{2}\lambda Q$ represents the deterministic NPV trigger value and $\frac{1}{\beta}$ is the term of adjust for the value of waiting. It confirms the initial remark that the optimal trigger value has to exceed $\frac{1}{2}\lambda Q$. It also demonstrates that the deterministic method is inappropriate for the estimation of an investment under uncertain circumstances. The optimal trigger timing to OMRs calculated by option pricing technique is higher than the trigger value derived by the deterministic pricing technique. Financially, we interpret this phenomenon as issuers not rushing to trigger their repurchase activity. They will wait longer to have a clearer picture of the uncertain phenomenon than to trigger their repurchase activity, so as to create the maximum profit for their long term shareholders. The complete model derivation and solution process are presented in the Appendix B.

Optimal repurchase volume

Eq. (5-8) is a derived expected revenue function with the assumption of a given repurchase amount. Depending on Eq. (5-8), we can further derive the optimal repurchase volume.

Differentiating Eq. (5-8) by Q to get the optimal repurchase volume, Q^{*} is equal to $\frac{g}{\lambda}$. We then follow the previous solution procedure to solve the optimal trigger timing with optimal trigger volume in a one-time repurchase model. The following optimal trigger timing results can be obtained:

$$g^{**} = \frac{2}{\beta} \tag{5-18}$$

[8]

[8] The variable which has the sub-notation ** represents the variable of optimal trigger timing with optimal repurchase volume

$$A=\frac{Ng^{**}}{\lambda}\frac{1}{\beta}e^{-\beta g^{**}}=\frac{2N}{\lambda\beta^2}e^{-2} \tag{5-19}$$

Substituting Eq. (5-18) into Eq. (5-19) and $Q^*=\frac{g}{\lambda}$ into Eq. (5-12), the value of the repurchase option for a optimal repurchase amount Q^* is given by

$$F(g)=\frac{2N}{\lambda\beta^2}e^{\beta g-2} \tag{5-20}$$

5.1.4 Discussion of results

In this section discuss the results intuitively and interpret the outcomes as well as any shortcomings/problems, one being the restriction to a single repurchase, which gives you the link to the following section

To verify the accuracy and rationality of the one-time repurchase model, we conduct a numerical analysis with the parameter setting as follows: annual risk free rate, ρ, is ranges from 0.001 to 0.2; liquidity parameter, λ, is 0.25; total issue shares, N, is 18,000,000; the standard deviation of rate of return, σ_s, ranges from 0.01to 0.5; and repurchase volume, Q, ranges from 0.001 to 0.1.

The results obtained by the optimal OMRs contingent value pricing model are presented in the following Figures 5-1 to 5-3, and these are consistent with the demonstrations in *chapter 4*.

The repurchase volume, Q, has a positive relationship with the optimal trigger value, g^*; The standard deviation of return, σ_s, has a positive relationship with the optimal trigger value, g^*; The interest rate, ρ, has a negative relationship with the optimal trigger value, g^*. The results produced by numerical analysis are reasonable and consistent with capital investment theory. In other words, a bigger investment risk combines a higher trigger point to yield a better contingent value; and a bigger risk free rate combines a bigger waiting cost so that issuers have to trigger earlier.

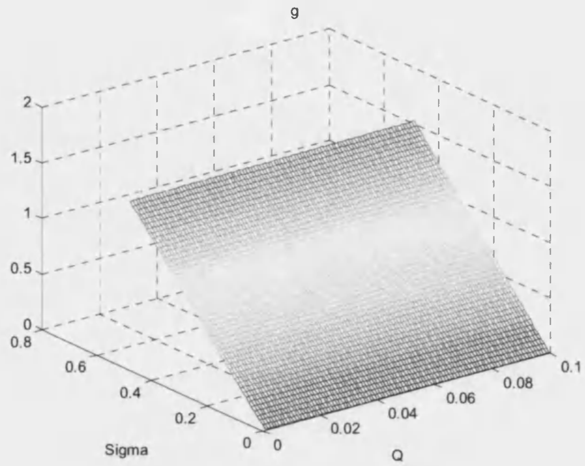


Figure5-1 Trigger value as a function of the repurchase amount and variance of return market price

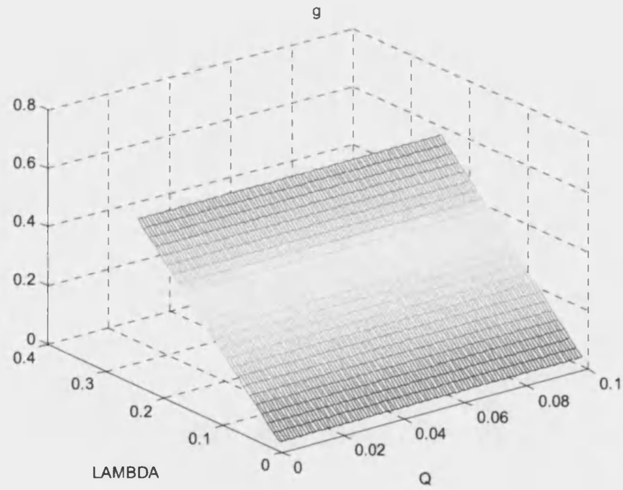


Figure5-2 Trigger value as a function of the repurchase volume and liquidity

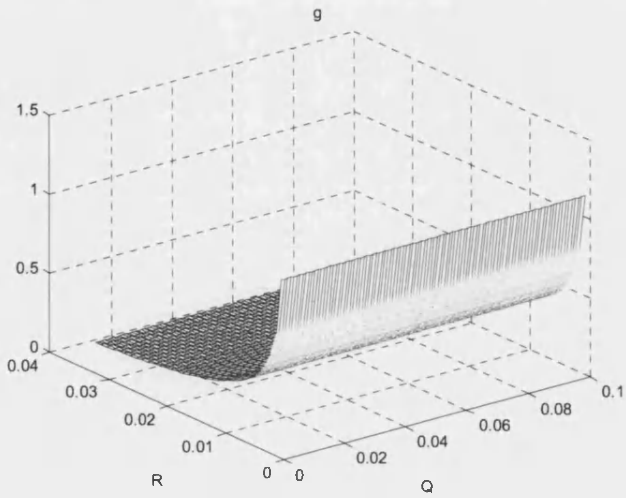


Figure5-3 Trigger value as a function of the repurchase amount and interest rates

Except for the above numerical illustration on the optimal trigger timing result, we can also depend on the auxiliary mathematical techniques to verify our numerical illustration. The partial derivatives of the option value have the following properties:

1. $\dfrac{\partial g^{*}}{\partial \sigma} > 0$ and $\dfrac{\partial g^{**}}{\partial \sigma} > 0$
2. $\dfrac{\partial g^{*}}{\partial \rho} < 0$ and $\dfrac{\partial g^{**}}{\partial \rho} < 0$
3. $\dfrac{\partial g^{*}}{\partial Q} > 0$

The details of mathematical proofs are presented at the Appendix C.

However, the real options model of one-time share repurchase timing exposes some drawbacks. For example, the one-time repurchase model cannot explain the realistic operation of OMRs which are conducted sequentially. Secondly, the one-time repurchase model has assumed that the repurchase action has no impact on the market price. Issuers take the whole risk to buy the shares and expect that the market price can revert to the issuers' expected prices. Based on the hypothesis that OMRs is a tool of investment (Vermaelen, 1981; 1984) and the hypothesis support for the share price (Ikenberry/Vermaelen, 1996), the one-time repurchase model fails to reveal that share repurchase programmes are able to generate a short term abnormal positive return/ or to salvage the undervalued share price. Therefore, there is a need to derive the sequential repurchase model.

5.2 Multiple share repurchases

This section will extend the one-time repurchase model to that of an optimal multiple sequential contingent value pricing model for OMRs. This extensive model development aims to derive a sequential repurchase model which is close to practical operations. Here, the issuers' trigger is at the optimal trigger timing while the repurchase volume is assumed to be fixed.

5.2.1 Assumptions of the model

OMRs are irreversible share buyback investment. Every trigger OMRs and cessation of the share buyback strategy incurs no extra cost because there is no threshold cost to enter OMRs and there is also no loss in ceasing the repurchase programme. Therefore, we can declare OMRs to be a sequential repurchase model with characteristics of irreversible investment, capacity choice, free threshold and free re-entry cost of capital investment programme.

To satisfy the volume condition, we assume that there is a fixed rate at which outlays and repurchase can proceed. $\bar{q}$ is the fixed rate at which the firm can purchase in a small time interval. $\bar{q}$ satisfies the constraint of permitted daily repurchase volume and the summed up $\bar{q}$ in each time period is small and equals to the total announced repurchase amount.

The repurchase mechanism, is such that when issuers decide to execute the repurchase programme, they repurchase periodically. However, if the future market price increases to reach their expected value, they suspend their repurchase activity. However, if the future market price continues to fall to a sufficiently low price, issuers can suspend the programme, until the market price starts to rise later, during which the issuer can resume stock repurchase at the point it previously left off.

In order to capture the above market share price change in the share buyback strategy, we need to employ a new variable , k, to assist our model setup. k is the total remaining expenditure required to complete the project. The dynamics of k are given by

$$dk = -\overline{q}dt \tag{5-21}$$

We therefore have two state variables that affect the optimal investment decision. The first is the remaining repurchase surplus to continue to proceed to the next repurchase, k. When k is equal to zero, it implies that the whole buyback programme is completed. The second is the asymmetry payoff price difference between the fundamental value and the market price, g_s, in the sequential repurchase model.

In this model, the firm's repurchase action, $d\eta$, is assumed to be effective in salvaging the undervalued market price. The market price follows a dynamic geometric Brown-Wiener process with the shock of share repurchases after first-time repurchase activity.[9]

$$d(\ln P) \approx \frac{dP}{P} = \frac{dV}{V} + \lambda\left(\frac{S}{N} + \frac{R}{N}\right)$$
$$= \mu_V dt + \sigma_V dz_V + \lambda \sigma_s dz_s + \lambda d\eta \qquad (5\text{-}22)$$
$$= \mu_V dt + \sigma_P dz_P + \lambda d\eta$$

where

R : represents total repurchase portion and it is equal to $\overline{q}N$

$d\eta = \overline{q}dt$

If the share repurchase is temporarily stopped, then $d\eta = 0$. We neglect the increase in the firm value due to buying undervalued shares as the amount is small relative to the value of the company.

Besides, the fundamental value is the same as Eq. (5-1), which follows a simple Brown-Wiener process.

$$dg = -\lambda \overline{q} dt + \lambda \sigma_s dz \qquad (5\text{-}23)$$

[9] The instantaneous market price change in the first time of sequential OMRs is the same as the change in the one-time repurchase model because there is only the disturbance of noise traders in the first time trigger. Besides, the issuer's repurchase is assumed to effectively salvage the undervalued price, so that the price dynamics has to take into account the issuers' efforts on repurchasing from now on.

Because there are no adjustment costs or other costs associated with changing the rate of investment, the optimal rate of investment will be either 0 or $\bar{q}$.

As always, the firm has an option to invest that it may or may not exercise. We will denote $F(g,k)$ as the value of this option, assuming it is exercised optimally, that is, assuming that the firm follows the optimal investment rule by repurchasing stock when $g \le g^*$, and not repurchasing otherwise. Then, as in the earlier section, we will find $F(g,k)$ and obtain the critical value of $g^*(k)$ as part of the solution. We can do this using dynamic programming.

5.2.2 Derivation of model results

As the explanation stated, two questions of interest to issuers are: firstly, what is the best price to trigger OMRs and how many shares they have to buy back when the OMRs is undertaken. To derive this optimal model, the two-dimension Taylor expansion and Ito's Lemma have to be employed. The optimal contingent value of OMSR is derived in the following Eq. (5-24). The details of the derivation of the diffusion process of the sequential repurchase model are presented in Appendix D.

$$\frac{1}{2}\lambda^2\sigma_s^2 F_{gg} - \lambda\bar{q}F_g - \bar{q}F_K - \rho F = 0 \tag{5-24}$$

As introduced in the previous section, k represents the remaining portion of stock to be repurchased. $\bar{q}F_K$ captures exactly the meaning of sequential repurchases/investment because the unrealized contingent value is equal to the marginal unfinished repurchase portion times the repurchase volume in that period. Mathematically, Eq. (5-18) is a partial differential equation, PDE, with given constant coefficients. There is no specific general solution to such kinds of partial differential equation. To solve the optimal contingent evaluation model, we need the extra conditions to define the stochastic diffusion of sequential repurchases.

There are three conditions to be taken into account: one of the conditions is an initial condition, which relates to the remaining repurchase portion, k. The other two conditions, which are boundary conditions, are about the asymmetry payoff, g. The first boundary condition we consider with respect to the trigger value is the same as Eq. (5-10). The second boundary condition which we consider with respect to the upper bound of trigger value, is as shown in Eq. (5-25). This condition is formulated by considering the rule of the details of open market stock issue. There is usually a limit to the lowest price of a company. When the stock is undervalued too much (larger than the threshold) it faces a take-over threat, $B(\bar{g})$ is then the profits the company makes from this take-over, which we assume to be constant, by our notation.

Boundary condition: $F(\bar{g},K)=B(\bar{g})$ (5-25)

Initial condition: $F(\bar{g},0)=0$ (5-26)

Next, regarding the initial condition, we consider that when K is equal to zero, then there is no option value that exists because the whole repurchase programme has been completed.

Employing the variable separation method to solve the general solution to Eq. (5-24), we can get the general solution of the optimal contingent value evaluation model. The details of the general solution of sequential repurchase model are presented in the Appendix E:

$$F(g,K)=B(\bar{g})e^{m_1(g-\bar{g})} \quad (5\text{-}27)$$

and

$$m_1 = \frac{\bar{q} + \sqrt{\bar{q}^2 + 2r\sigma_s^2}}{\lambda\sigma_s^2} \tag{5-28}$$

Eq. (5-27) and Eq. (5-28) reveal an unexpected result at first glance. The un-repurchased share has no influence on the general solution of the sequential repurchase model with respect to the remaining share repurchase activity. We can conclude two potential reasons upon further analysis: First, there is no extra adjustment cost with respect to the repurchase volume at each stage. It is constant irregardless of the cost per share. As regards the average repurchase portion, $\bar{q}$, is employed because the open market is an uncertain market. Taking the average the repurchase portion helps eliminate the risk of non-constant repurchase volume. The concept is the same as that of the least square method which reduces the estimation error. Both reasons lead to an independent repurchase activity at each stage, thus the un-repurchased portion, k, has no effect on the contingent value in a sequential repurchase process.

As regards the boundary conditions for optimization purposes, the value-matching condition and smooth-pasting condition are required to find the critical point as well.

To realize the issuers' expected sequential revenue, we need the condition of the revenue function to be tangent with the diffusion curve at each time period. When the tangent point is found, then we can be assured that the optimal trigger point for sequential repurchase model is found.

The derivation of expected revenue function is derived as follows. The repurchase time is divided into n subintervals of equal length. Hence, the expected revenue function at the first stage is derived as follows:

1st-stage:

$$\pi_1 = 0 + N\int_{T}^{T+\frac{1}{n}}\left(\ln V_s^1 - \ln P_s^1\right)d\eta = \overline{q}N\left(g_s - n\times\frac{1}{2n^2}\lambda\overline{q}\right) = \overline{q}N\left(g_s - 1\times\frac{1}{2\times 1}\lambda\overline{q}\right)$$

The repurchase activity is assumed to effectively salvage the undervalued market price. Based on the theory of Martingale property, the expected trigger value at the second stage is equal to the expected trigger value at the first stage which deduces the shock of the second time repurchase influence in the second stage.

The expected revenue function at the second stage is derived as follows:

ith-stage:

$$
\begin{aligned}
\pi_i &= \bar{q}N\left(g-\frac{(i-1)\lambda\bar{q}}{2n^2}+\left[\int_{T+\frac{i}{n}}^{T+\frac{i+1}{n}}\left(\ln V^t-\ln P^t\right)d\eta\right]\right)\\
&= \bar{q}N\left(g-\frac{(i-1)\lambda\bar{q}}{2n^2}-\frac{\lambda\bar{q}}{2n^2}\right)
\end{aligned}
$$

Inferring the issuer's expected revenue to the n[th]-stage:

$$
\pi_n = \bar{q}N\left(g-\frac{(n-1)\lambda\bar{q}}{2n^2}+\frac{\lambda\bar{q}}{2n^2}\right)=\bar{q}N\left(g-\frac{\lambda\bar{q}}{2n}\right) \tag{5-29}
$$

As regards the boundary conditions for optimization purposes, the value-matching condition and smooth-pasting condition are required to find the critical point as well.

Finally, we need a smooth-pasting condition which we obtain from the value-matching condition above as follows:

$$
\pi_g\left(g^*,T\right)=\bar{q}N \tag{5-30}
$$

5.2.3 Solutions of Process and Results

We can solve the partial differential equation under the imposed boundary conditions of Eqs. (5-29), (5-30) and (5-10) and using the general solution for Eq. (5-27).

Differentiating Eq. (5-21) by g and defining g^* as the point tangent with the n[th] expected revenue function, we can generate the coefficient of general solution $B(\overline{g})$ as follows:

$$B(\overline{g}) = \frac{\overline{q}N}{m_1 e^{m_1(g^* - \overline{g})}} \tag{5-31}$$

Substituting Eq. (5-31) into Eq. (5-27) and equating the final equation to the n[th] expected revenue function, we obtain the optimal trigger value of the n[th] expected revenue function.

$$F(g^*, K) = \frac{\overline{q}N}{m_1 e^{m_1(g^* - \overline{g})}} e^{m_1(g^* - \overline{g})} = \overline{q}N\left(g^* - n \times \frac{\lambda \overline{q}}{2n^2}\right)$$

then

$$g_s^* = \frac{1}{m_1} + \frac{\lambda \overline{q}}{2n} \tag{5-32}$$

$$F^*(g_s^*, K) = \frac{\overline{q}N}{m_1} \tag{5-33}$$

Finally, we substitute Eq. (5-31) and (5-32) into Eq. (5-27), and arrive at the general diffusion equation of the n^th^-stage repurchase model, as shown below:

$$F = Ae^{m_1(g-\bar{g})} = \frac{\bar{q}N}{m_1 e^{m_1\left(g^* - \bar{g}\right)}} e^{m_1(g-\bar{g})} = \frac{\bar{q}N}{m_1} e^{m_1\left(g - \left(\frac{1}{m_1} + n \times \frac{\lambda \bar{q}}{2n^2}\right)\right)}$$
$$= \frac{\bar{q}N}{m_1} e^{m_1\left(g - \frac{\lambda \bar{q}}{2n}\right) - 1} \tag{5-34}$$

Eq. (5-31) is similar to the solved optimal trigger timing, Eq. (5-14), of the one-time repurchase model; the optimal trigger value has to exceed $\frac{1}{2}\lambda\bar{q}$ and it is consistent with the previous demonstration of the deterministic method which is inappropriate for the estimation of an investment under uncertain circumstances. However, there is a difference between Eq. (5-28) and Eq. (5-13) which lies in the parameters m and β. This is due to the effect of the repurchase on prices and thus on the profits of future repurchases. In other words, repurchase activity is assumed to effectively exploit the undervaluation of the shares through its optimal timing. As a consequence of the repurchase activity the difference between fundamental value and market price reduces gradually. The theoretical optimal value in a sequentially repurchase model is getting smaller. Eq. (5-32) just explains this theoretical expectation. More clear numerical illustrations are presented as follows under the discussion of results.

5.2.4 Discussion of results

The optimal solution reveals the same result that optimal trigger timing and optimal contingent value do not appear to depend on k; it depends on the maximum repurchase rate $\bar{q}$ at each time interval only. However, there is a slight relationship between k and $\bar{q}$. Before issuers take any repurchase activity, k is equal to the intended buyback volume, and $\bar{q}$ is equal to $\frac{Q}{n}$. The relationship is built on here.

Having now established the optimal trigger timing model and contingent value of the option to repurchase stocks, we can now continue our sensitivity analysis to determine the optimal trigger value g^* at which the option is to be exercised.

As is obvious from the inspection of Eq. (5-26) above, the trigger value is a linear combination which increases with the liquidity of the market, as confirmed in Figure-5-1. Intuitively we would expect the trigger value to increase in the repurchase amount as the larger repurchase would drive up the price, thus decreasing the profits available. Inspection of Figure- 5-4 proves the above expectation.

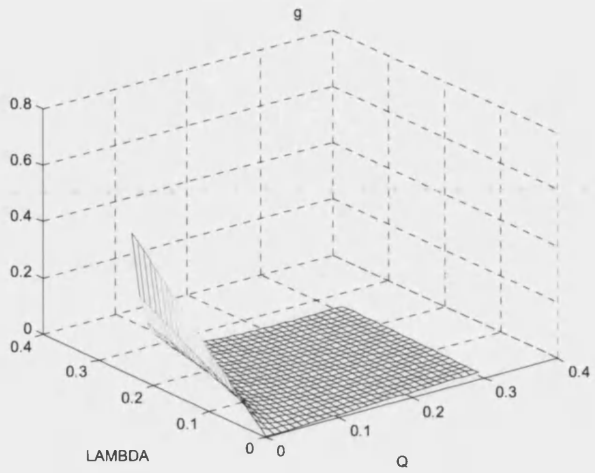


Figure 5-4 Trigger value as a function of the repurchase volume and liquidity

We can explain this finding by considering the profits the company would make from the repurchase as defined by its objective function[10]. When repurchasing a small amount, the profits will be relatively small. Thus, when the values differ only marginally, it is beneficial to wait for a larger difference as this would translate to a higher profit. As the repurchase amount increases, the profits also increase accordingly, thus the incentives to conduct a repurchase also increase. It can actually be shown that the profits from the repurchase are strictly increasing in the repurchase amount. As the repurchase amount increases, the price effect of the transaction will become more and more important until it finally dominates and the trigger value increases.

[10] We thank Richard Fairchild for providing us with some insights into this relationship

The explanation of volatility in Figure 5-5 is the same as the explanation in the one-time repurchase model. The influence of the repurchase amount is the same as the explanation in Figure 5-4. We would not repeat it here.

Figure 5-6 reveals the relationship amongst optimal trigger value, repurchase amount, and interest rates. This figure reveals that the interest rate has a negative non-linear relationship with trigger value and it is consistent with Eq. (5-32).

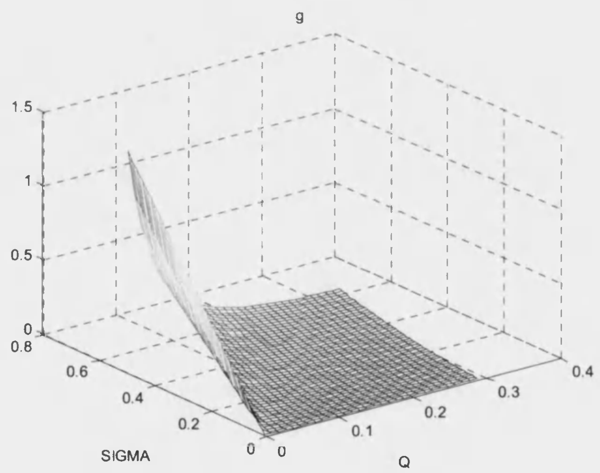


Figure5-5 Trigger value as a function of the repurchase volume and variance of return

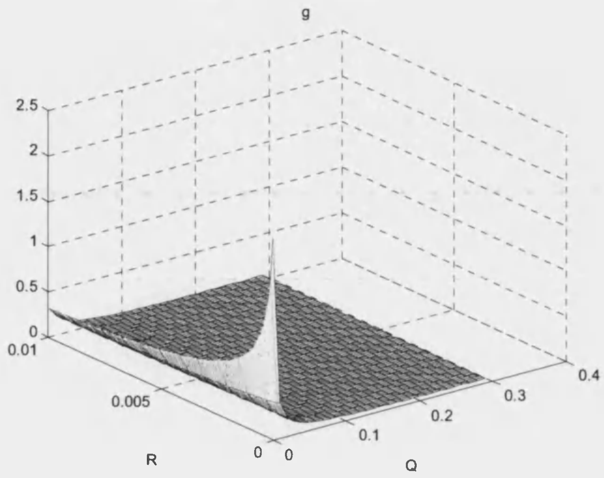


Figure 5-6 Trigger value as a function of the repurchase volume and the interest rate

5.3 Empirical implications

The next chapter will conduct an empirical investigation. This empirical study aims to further test the validity of the developed optimal trigger timing models using real data. With this confirmatory study, we can then use our optimal trigger models to criticize practical OMRs' operative performance. The empirical implication starts with a hypothesis test on both the one-time repurchase model and sequential repurchase model. Then the comparison between practical repurchase performance and theoretical optimal trigger timing will be undertaken.

5.3.1 One time repurchase model:

Hypothesis 1: The optimal trigger timing has a positive relationship with the standard deviation of return, σ_s. *A bigger repurchase volumeresults ins a higher trigger point.*

This hypothesis aims to verify that, if a company's share return is volatile, then the issuer would not rush to trigger their repurchase activity. Their repurchase strategy is to wait longer for the appropriate time to capture higher returns for their long-term shareholders..

5.3.2 Sequential repurchase model:

Hypothesis 2: With multiple repurchases, the thresholdfor conducting OMRs is higher for the first repurchase than for subsequent repurchases.

In an incomplete market, the price of goods will change when a new market participant enters. This price change results from demand –supply changes. After that, the new equivalent point will stabilise. The further market price will not change sharply as long as no new market participant enters. Our sequential repurchase behavior assumes similar economic behaviour. Before the first-time trigger, the market has not experienced a repurchase impact. When the first –time trigger takes place, the market participants change so that the price difference changes as well. But the further repurchase activity will not change the price difference dramatically. Thus, there should be a kink between first time trigger and the second time trigger.

Hypothesis 3: The repurchase acvtivities are timed such that they generate profits for the company which are reflected in a higher stock price after the repurchase.

This is an important assumption in the sequential repurchase model. However, from the numerical analysis in the Section 5-2, the small repurchase volume is not the case. This hypothesis aims to confirm the validity of numerical analysis in Section 5-2.

5.4 Limitations of the developed models

This study pioneers to develop the optimal trigger timing and optimal volume models to issuers of OMRs. There are several limitations which challenge our research results. First of all, OMRs are embodied as a non-commitment repurchase programme so it is difficult to evaluate the total contingent value of executing OMRs. The second limitation is that it is difficult to evaluate the signalling effect. The final limitation is the concern with respect to the mathematical technique used for the model setup and solution processes.

As regards the first limitations, OMRs is not an obligation to invest. Companies hold the rights to repurchase but have no obligation to complete the pre-announced repurchase programme. The flexibilities of OMRs makes it difficult to accurately evaluate other documented revenues with respect to an uncertain repurchase volume and repurchase price. This limitation results in the sequential model being unable to accurately evaluate the total investment cost and the contingent value from the execution of OMRs.

With respect to the signalling effect of OMRs, Ikenberry/ Vermaelen (1996) and McNally (1999) pointed out that the explanatory variables to signalling OMRs include company size, reputation, economic growth, returns etc. and that they are able to influence the magnitude of signalling effect. In other words, the signalling effect could vary between companies. Even the same company could have different signalling effects in different time frames so that it is hard to accurately evaluate its contribution into our developed model.

Finally, we assume that executing OMRs is unlimited time so that the time value of the contingent value of OMRs is ignored. Further study can develop an option with a maturity time frame thus incorporating time effect into the optimal trigger timing models. However, this requires a more complicated mathematical technique and more boundary conditions to assist the derivation of the optimal repurchase trigger timing model. Unfortunately, it is hard to provide boundary conditions to this type of model. This problem is also mentioned by Merton (1968).

Chapter 6
Empirical investigation

6.1 Introduction

A company's fundamental value is an important parameter with respect to the developed optimal trigger timing models. Ikenberry/Vermalen (1996) and this study declared that fundamental value determines the options values of OMRs. It is also a parameter to test whether the operations are practical to an empirical study. In the theory of optimality, it is a factor to decide the optimal trigger price of OMRs. In other words, it is not only a parameter for the estimation of optimal trigger timing but it is also a parameter to test whether the operation is optimally practical. Prior to undertaking this empirical investigation, the estimation of fundamental value will be introduced. Once the estimation of a company's fundamental value has been studied, then the following empirical investigation can be undertaken.

We test the developed models and practical verification with the hypotheses mentioned in *Chapter 5* in this chapter. The investigated companies are a sample selection from a single industrial sector (information technology). We conduct descriptive statistics aimed at important parameters and then prepare the data for further extensive study uses and conduct several simple analyses for practical OMRs operations. Next, we substitute input parameters into the developed models to analyse the theoretically optimal trigger timing and to analyse the differences among different optimal trigger timing models of OMRs. Furthermore, by analyzing the difference between theoretical and practical trigger timings, we will provide the optimal repurchase strategy for practical operation purposes which will be recommended at the end of this section. This empirical investigation aims not only to verify the accuracy and applicability of the developed models but also to provide recommendations to conduct OMRs for purposes of practical operations and to respond to arguments in the existing literature.

In Section 6.2, we introduce fundamental value analysis. In Section 6.3, we examine the sampled companies data that affects the decision to initiate a share repurchase.

Section 6-4 undertakes the analysis of real options models of share repurchase timing. Here we also conduct a series of tests relating to both the theoretical optimal repurchase strategy and practical operations. Section 6-5 details a final summary of our empirical study. Section 6-6 provides a critical analysis on this empirical study.

6.2 Estimation approach of fundamental value

There are two different estimation methods which will be employed to estimate the fundamental value; one is the practical method and the other is derived by the capital pricing theory. With the objective and accurate fundamental value appraisal methods, we are able to transform the practical operation into a testable index.

6.2.1 Practical estimation method

The first method is widely used in financial practices Taiwan. This method multiplies the financial index of price-earnings ratio by the earnings per-share of a firm. Mathematically, this can be represented as

$$V = \frac{P}{E} \times EPS \qquad (6\text{-}1)$$

$\frac{P}{E}$: price to earnings ratio

EPS : earning per share

The price to earnings ratio assumes that the corporation will be worth some multiple of its future earnings. A higher price to earnings ratio is often associated with growth stocks, or companies that are growing faster than average. It is a well used indicator to assist analysts in the estimation of a company's value. (Value Based Management.net, 2005). In line with common business practice in Taiwan, we assume that a firm's fundamental value in 2004 is equal to its price to earnings ratio averaged over 2004 multiplied by the forecast of their 2004 earnings per share. However, the estimation of Eq. (6-1) is easily manipulated by a volatile financial index. To avoid using an improper and too sensitive an index the data of which could be too sensitive for the evaluation of fundamental value, we adopt the average past four years price-to-earnings ratio to moderate any single year abnormal data.

Taking price-earnings ratio of an index instead of a single company offers a different explanation. If their price earnings ratios are below the industry average, issuers believe that their companies are undervalued. Issuers also believe that their cumulative value is worthier than the current market price. Therefore, taking the average price to earnings index multiplied by the 2004 earnings per share can take their past accumulated average performance and future prospective earnings into account.

6.2.2 Theoretical estimation method

The second method of estimating fundamental value is based on stock valuation theory. The stock price of an investment can be derived as follows:

$$P_0 = E\left(\frac{D_1 + P_1}{r}\right) \text{ or } P_0 = E\left(\frac{D_1}{r}\right) + E\left(\frac{P_1}{r}\right)$$

Where

r: expected growth return on the stock investment of one time period. (expected capitialisation rate)

D_1: flow payoff (dividend)

The current stock price is equal to the expected stock price in next period plus flow payoff (dividend to shareholders) –which discounts for time. Assuming the flow return is permanent with a growth rate of c, and assumed to hold the stock permanently, we then can rewrite the above stock evaluation formula as Eq. (6-2)

$$\begin{aligned} P_0 &= E\left(\frac{D_1}{r}\right) + E\left(\frac{P_1}{r}\right) \\ &= E\left(\frac{D_1}{r}\right) + E\left(\frac{cD_1}{r^2}\right) + E\left(\frac{c^2 D_1}{r^3}\right) + \ldots.. + E\left(\frac{c^i D_1}{r^i}\right) + E\left(\frac{P_i}{r^i}\right) \end{aligned} \quad ; \; i \to \infty \qquad (6\text{-}2)$$

We then can re-arranged Eq. (6-3) as Eq. (6-3):

$$P = \frac{D_1}{r - c} \qquad (6\text{-}3)$$

Eq. (6.3) is the famous stock pricing model, the Gordon formula. In that model, the market asset price is assumed to be equal to the fundamental value of the asset so that the market price is equal to its fundamental value. However, this hypothesis of efficient market is unsuitable for our study purposes because the market price is undervalued in a bear market (Craine, 2005), so that a company's fundamental value will not be equal to its outstanding share price. In other words, dividend is irrelevant to the estimation of fundamental value in the inefficient / imperfect market.

To have the objective estimation method of fundamental value, we employ the principle of Eq.(6-3) and change the required variables to be as follows:

$$V = \frac{EPS}{H - c} \tag{6-4}$$

A company's fundamental value is based on a company's future (discounted) cash flow.The net earning growth return of assets, $H - c$, represents a firm's ability to grow (earning growth return of assets minus the potentially payout ratio). Taking EPS divided by net earning growth return of assets, the fundamental value can be obtained. In order to derive this formula we replace dividends in equation (6-3) by EPS, which can be justified by the Modigliani-Miller hypothesis on the irrelevance of dividends.

The arguments to employ EPS instead of adopting dividend in Eq. (6-4) include: First, dividend represents the flow payoff of investing in a stock, and EPS represents the operation profit of an asset. Using EPS to estimate a company's fundamental value is thus reasonable. In addition, using dividends to estimate a company's fundamental value is far-fetched because any dividend issue relates to a company's disposable money distribution method to its shareholder. Employing this index to estimate a company's fundamental value might create bias in the estimation of a company's fundamental value. Hence, we employ EPS to estimate a company's fundamental value.

The EPS employed here is the financial index of 2004 earnings per share. To avoid volatile financial statistics, H is the average past four years growth return of total assets. Taking the index of average rate of return of total assets instead of a single year index can avoid the volatile index in a single year, so as to increase the accuracy of the estimated fundamental value.

6.3 Data description

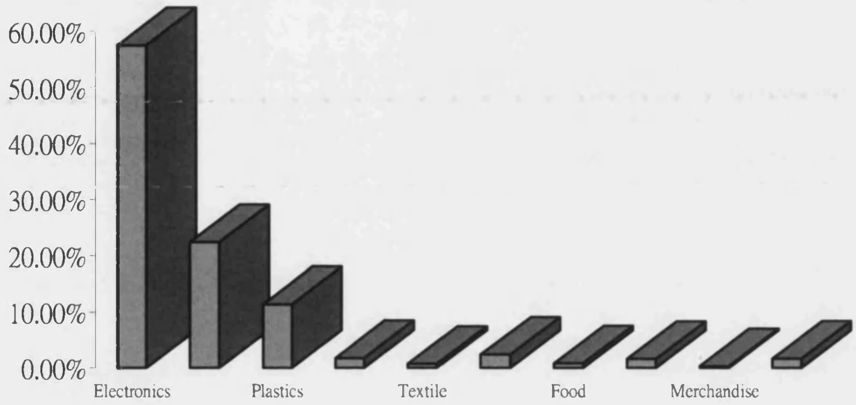
6.3.1 Sample selection

Our developed models aim to create an optimal trigger timing models for issuers, so as to assist them to reach maximum profit for their long-term shareholders. Therefore, all the sampled companies in the empirical investigation have to launch their repurchase programmes for the purpose of investment as they repurchase outstanding shares with the aim of maintaining a company's credit worthiness and their shareholder wealth.

The sampled companies in this empirical study are those companies sampled within the FTSE Taiwan 50. The index of FTSE Taiwan 50 is calculated by the average weighted 50 public companies. The total value of these companies is over 70% of the aggregate Taiwan market capitalization. It is one of the most representative economic indices in Taiwan. FTSE Taiwan 50 is comprised eleven industries which include Electronics, Banking and Insurance, Plastics, Telecom, Textile, Steel Structure, Food, Motor, Merchandise, Transportation, and Others.

The Taiwanese electronics industry (or IT industry) is the most representative industry. The product value of Taiwan IT industry is over 8% of Taiwan GDP and over 46% of IT accessories were exported from Taiwan in 2003. The market capitalisation of the IT industry is over 57.51% which is over half the market capitalization of the FTSE Taiwan 50 (see Figure 6-1). The majority of these electronics companies have executed OMRs, therefore, we will investigate the sampled Taiwanese Electronic companies among the FTSE Taiwan 50 in this study.

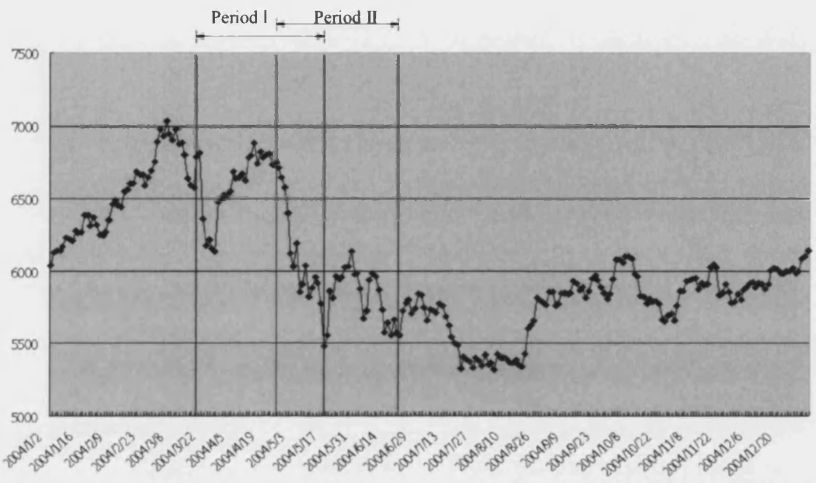
Based on the Taiwan Stock Exchange Corporate (TSEC) 2004 record, there are nine companies which executed OMRs. However, two of these companies have negative earnings per share and they are unsatisfactory for our purpose of empirical verification, so we have seven companies remaining which will be our investigative objects. The final chosen companies are Acer, Compal, MTK, Quanta, Realtek, TSMC, and UMC.



This figure shows the distribution of the industrial capitalization of FTSE Taiwan 50 in 2004 which is mainly catalogued as 11 industries. The Electronics industry occupies over half of the total market capitalisation of FTSE Taiwan 50 (SinoPac, 2004).

Figure 6-1 The distribution of industrial capitalization of FTSE Taiwan 50

Their repurchase schedule concentrates on two periods that are end of March to end of May and the beginning of May to beginning of July for a total period of two months. During this period, there is a sharp decline around end of March and there is a further decline which broke through the 6000 point of the Taiwan weighted stock index. To maintain a company's creditworthiness and the rights of shareholders, our sampled companies executed OMRs to salvage the undervalued price. Some of these sampled companies executed their repurchase programmes more than once during 2004 such as UMC which executed OMRs twice in 2004. We select the first-time repurchase programme to form our empirical verification objective. Figure 6-2 shows the Taiwan weighted stock index in 2004; both repurchase periods are apparently during the bear market.



This figure reveals two periods of stock recession in 2004 in Taiwan; one from end March and the other from the beginning of May. In the first time recession, Acer, Compal, TSMC, and UMC executed OMRs. MTK, Quanta and Realtek executed OMRs in the second time recession.

Figure 6-2 Taiwan weighted stock index in 2004

6.3.2 Data collection

To verify the validity of our study and conduct a practical comparison, the data collection task includes collecting: (i) the OMRs information of the sampled companies (ii) the data for the estimation of fundamental value and (iii) the input parameters for the application of our developed models. Our initial data collection of share buyback is obtained from TSEC and supplemented with details from the Taiwan Economic Journal Database, TEJD.

1. Data Collections of and the relative OMRs information of the sampled company

The task in this part includes collecting the basic information of a company and the realised repurchased information. The basic information includes the company's name, the major industrial product and the total number of issued outstanding shares (N), repurchase fraction (Q), and the average price during the repurchase period (AP).

The realized information includes the repurchase fractions/volume, repurchases frequency, average repurchase price (ARP) and the average repurchase volume (ARV). The realized information is pending our verification to determine if these companies have indeed repurchased their shares in a manner that is beneficial to shareholders.

2. Data Collections for estimating fundamental value

There are three parameters for estimating fundamental value: earnings per share , EPS , the net growth rate of total assets, denoted as $(H-c)$, and the P/E ratio. To avoid the volatile single financial index which results in an unrealistic estimation, the net growth rate of total assets $(H-c)$ adopted are those of the past four years average data; and P/E represents the individual company's average P/E value in the past four years. EPS is taken from the 2004 record in 2004 on an after-tax basis. It is taken with an after tax adjustment. With respect to the net growth rate, $H-\rho$ is defined as per the previous section; H is calculated from the average past four years growth rate of the total assets of the sampled companies.

3. Data Collections for Model Set-up

The required input variables for the empirical test for our one-time repurchase model include N, Q , ρ, σ_s and λ. Among the above parameters, N and Q have been introduced in the aforementioned company's information. The risk free rate, ρ, is the savings interest rate of the Bank of Taiwan as issued in 2004. The value of the standard deviation of the rate of return, is denoted as, σ_s. As regards market liquidity, λ, we draw reference from Lin (2003) and take the statistics of a bear market because we assume that OMRs are executed in a bear market, especially, when issuers perceive OMRs as an investment tool and the bear market scenario is an appropriate time to execute such OMRs.

As regards the required input variables for the empirical test of the optimal sequential repurchases model, these are more complex than those of the one-time repurchase model. As mentioned in *Chapter* 5, there are another three input variables, repurchase frequency, the maximum repurchase rate in a time interval, and critical asymmetry payoff, which are required to be employed to calculate the theoretical optimal sequential repurchase timing. Among the above extra input variables, the critical asymmetry payoff, is calculated by taking the natural log of the critical pricing difference between fundamental value and market price; the buyback portion at each stage is assumed such that issuers repurchase a constant portion of outstanding shares which is equal to the ratio of the total repurchase portion over the repurchase frequency. It varies with each individual company's buyback volume and buyback frequency. The final input parameters- repurchase frequency, is set by a company's realised number of times of share repurchase.

6.3.3 Descriptive statistics

The descriptive statistics aim to conduct a simple analysis on the practical operations where issuers' average repurchase strategy such as repurchase price and repurchase volume will be analysed. Depending on the analysis of the average repurchase price and the repurchase volume, we can initially comment on their repurchase strategies. Besides, the index of the ratio of fundamental value divided by the average share price will also be discussed. Prior to analysing the above index, the fundamental value also has to be estimated.

1. Repurchases volume and repurchase price

Table 6-1 reveals that all investigated companies realised 3% below the number of shares to be repurchased. All the sampled companies repurchased a certain volume under the volume condition, which is no more than 10% of the total issued shares. Acer repurchased the maximum fraction of 2.583%, while Realtek repurchased the minimum fraction of 0.207%, as compared to the other sample companies.

Regarding repurchase price, in order to analyse the issuers' performance of repurchase price, we employ another index – the average share price during the repurchase period. Comparing the average share price and the average repurchase price, the average repurchase price of each company is below its individual average share price during the repurchase period. The average repurchase price is between 90% and 99% of its average share prices. This basic analysis is able to explain that the issuer's repurchase strategy of buyback is intuitionally acceptable. In order to have a deeper understanding of practical OMRs operation, we will employ the traditional statistical method before beginning our series of discussions.

2. Fundamental Value

In order to estimate fundamental value, there are three parameters which need to be employed: price earning ratio, earning per share and net grow return of total asset. Table 6-1 shows that there is a different price earning ratio between different industrial sectors but there is a similar price earning ratio within the same industrial sub-sector. For instance, in the NB OEM sub-sector, Compal and Quanta have close price earning ratios of 11.02 and 13.31 respectively while TSMC and UMC also have close price earning ratios of 22.99 and 24.65 respectively in the IC design and semiconductor manufacturing sub-sector. A stock which has a higher price earning

Table 6-1 Basic Information of Sampled Companies

The following table provides summary information for open market share repurchase which includes the basic financial data and the relative data for the estimation of fundamental value. ρ is the one-year savings interest rate published by the bank of Taiwan. The other variable is collected from TSEC and TEJ.

Ticker symbol	Name of company	Industry Catalgoue	Issued Shares (N)	Repurchased fraction	Average price during repurchase period(AP)	Frequency	Average repruchase price (ARP)	Standard derivation of return	Average repruchase volume (ARV)	EPS 04	P/E ratio	Net growth rate(G-c)	Fundamental value (V1)	Fundamental value (V2)
Acer	Acer Ltd.	Computer system	2315014	2.583%	49.11	7	49	0.172	0.369%	3.75	19.67	9.810%	73.7625	38.23
Compal	Compal Electronics, Inc.	NB	3338261	2.074%	40.39	7	38.93	0.19	0.296%	3.08	11.02	3.925%	33.9416	78.47
MTK	Media Tek Inc	IC design	769336	0.388%	283.43	2	274.7	0.189	0.194%	22.33	17.58	5.033%	392.5614	443.72
Quanata	Quanta computer Ltd	NB	2315014	0.271%	69.76	1	65.68	0.186	0.271%	4.68	13.31	3.502%	62.2908	133.63
Realtek	Realtek Semiconductor Corp.	Network communication	7003684	0.207%	41.65	2	41.22	0.216	0.104%	3.38	21	4.367%	70.98	77.40
TSMC	Taiwan Semiconductor Manufacturing Company	IC design & Semiconductor	23251964	0.489%	59.03	14	57.86	0.179	0.035%	3.45	24.65	4.220%	85.0425	81.75
UMC	United Microelectronics coporation	IC design & Semiconductor	17791982	1.080%	30.13	8	27.1	0.189	0.135%	1.89	22.99	3.870%	43.4511	48.84

ratio is often associated with growth stocks, or companies that are growing faster than average. This argument is also shown in Table 6-1 and it is consistent with practical expected industrial value. IC design and the manufacture of semiconductor are identified here as growth stocks as they have higher earnings ability compared to NB OEM.

Besides, all the sampled companies have a positive earning per share. Among them, MTK has the highest earning per share at 22.3. As regards the net growth rate of total asset, Acer has a distinguished growth rate at 12.73%. We found that Acer merged one of its controlled companies in 2001. Therefore, we take the growth rate of total assets of Acer only from 2002 to 2004 instead.

Next, substituting the input parameters into the estimation methods of fundamental value, the different fundamentals can be obtained. The first fundamental value is calculated using a well used Taiwanese practice denoted as V_1. The second fundamental value is calculated by theoretical derivation, denoted as V_2. All estimated fundamental values are presented in Table 6-1.

Analyzing the estimated fundamental values, there are three estimated value are very different from each other that they are the estimated Acer's fundamental value by the theoretical estimation, Compal's and Quanta's have unrealistic estimated fundamental value by the practical estimation method. Their estimated fundamental values are lower than their average repurchase prices. This result shows that the real fundamental estimation method is invalid for the estimation of a company's fundamental value when it has a lower price earning ratio, such as in the case of Campal and Quanta. The theoretical estimation method has no such problem dealing with the estimation of firm value with a lower price earning ratio. Nevertheless, the theoretical fundamental value estimation method could also be biased when estimating a firm's value when it is experiencing sharp growth of its total assets. Acer is one such case. Acer has a quick growth of total assets because of the merger with one of its sub-companies in 2002. Though we have adopted the average method and tried to mitigate such an abnormal asset growth rate, the estimated fundamental value is still worse. In the following section, we only use the estimated fundamental value closer the actual price.

Except for those three estimated fundamental values, the other real trigger timings are defined as their individual fundamental values divided by their respective average repurchase prices, which are between 1.41~2.00. In spite of the discrepancy it still remains a practical consideration to launch a OMRs. Besides, as outsiders we are still unable to discuss which method (i.e. practical or theoretical) is better, the issuers know which method is more suitable for estimation of their own company's fundamental value because of intimate knowledge of the respective company. We provide two different methods for consideration and subject them to further discussion and analysis.

6.4 Data analysis

The following data analysis is organised into five parts. The first part studies the optimal trigger timings of one-time repurchase models under a given repurchase volume and test whether the sampled companies operated OMRs optimally. The second part tests the hypotheses which have been discussed in Section 5-4. This test will reconfirm whether our developed models are consistent with capital investment theory and numerical results by investigating real data. The third part studies the optimal trigger timings of one-time repurchase models with optimal repurchase volume and test whether sampled companies operated OMRs optimally. In the fourth part, we analyse the sequential repurchase model. We will also analyse the difference between the one-time repurchase model and the sequential repurchase model. In the fifth part, we further conduct an extensive analysis to prove that OMRs can be a useful investment tool in a bear market. The study in this part aims to verify our research results as to whether it is consistent with the existing reference's argument.

6.4.1 Optimal trigger timing study- one-time repurchase under given repurchase volume

1. Estimation of Optimal trigger timing

We analyse our optimal one-time repurchase models by substituting the required input data. The analysed results are presented in Table 6-2.

[Insert Table6-2 about here]

In the case of the optimal one-time repurchase model under a given repurchase volume, the optimal trigger timings, g^* , of the sampled companies are quite close, they are between 0.541~0.676. This value indicates that a company's optimal trigger timing holds when the market price falls down to a certain fraction of its fundamental value. Among the sampled companies, Realtek has the maximum trigger timing of 0.676. In other words, Realtek has to repurchase its outstanding share at the value when the market price is equal to $\frac{1}{1.966}$ of fundamental value to create the maximum profit for its long-term shareholders. Similarly, we found that Acer has the minimum trigger value of 0.541. In other words, Acer has to repurchase its outstanding share at the value when the market price is equal to $\frac{1}{1.718}$ of its fundamental value to reach its optimal repurchase timing strategy.

Incorporating both indices of the optimal trigger values and each sampled company's volatility of stock return to conduct a cross analysis, the optimal trigger value has a positive relationship with its volatility of stock returns. In other words, a company with a bigger trigger value has to wait longer to catch up with higher contingent value. This verification of the studied optimal trigger timing is consistent with the numerical test of *Chapter* 5 and the existing investment theory by Copland/Antikarov (2000).

2. Test of joint repurchase performance under different fundamental values

Having the optimal trigger timing of each sampled company, we can now study whether the practical repurchase performance is consistent with the theoretical optimal trigger timing. However, the above optimal trigger timing result is indirect result. Issuers/ Investors have to refer to their fundamental values to re-calculate the optimal trigger prices. Hence, we need to employ the estimated company's fundamental value to compute practical repurchase timings. Since different fundamental values studied and created, each company has two trigger values. Both the different trigger timings of each sampled company will be studied. This test is designed to test whether the practical repurchase value is consistent with the theoretically optimal trigger value.

The test is conducted by employing traditional statistic test methods to test their joint operative performances. We first take the natural log of the first type fundamental value divided by the average repurchase price, denoted as g_1, and the index of natural log of the second type fundamental value divided by each practical average repurchase price, denoted as g_2 [11]. Before conducting the joint test of optimality, each sample company's repurchase timing when calculated with the first type of fundamental value is lower than the optimal trigger timing. Thus, all sampled companies had triggered their OMRs too late, so that their market prices are undervalued for too long a time. They potentially exposed their companies to the threat of take-over. Similarly, we can analyse the sampled companies' repurchase timing when calculated with the second type of fundamental value. Here, companies

[11] The upper sub-notation represents different sampled companies, the lower sub-notation represents the practical trigger timing calculated by different fundamental value estimation

trigger too early such as Compal, MTK, Quanata, Realtek, TSMC and UMC.

As for the joint consistency test between the optimal trigger value and the first type practical trigger value and to the second type practical trigger value, the test results are as presented in Table 6-2. The first type of joint consistency test rejects the hypothesis that practical OMRs operated optimally with 95% significance interval. That means that if the first type fundamental value is objective, then our sampled companies did not execute their OMRs optimally. Their repurchase strategies remain to be improved. Conversely, the joint consistent test between optimal trigger value and the second type practical trigger value didn't reject the hypothesis that the practical OMRs have been operated optimally. In other words, if the second type fundamental value is consistent with the real fundamental value of sampled companies, we can then be assured that the joint practical repurchase performance is within significance interval, i.e., the sampled companies repurchase optimally.

Unfortunately, we are unable to get a consistent test result to comment whether the sampled companies repurchased their outstanding stocks optimally by referring to their different calculated fundamental values. The first type of fundamental value and the second type of fundamental value provide contradictory test results upon diagnosis. This conclusion is only right because we have found that both estimated fundamental values are very different and thus have different joint repurchase test results. Conversely, this contradictory test result responds to our description section 6.1 and the declaration by Ikerberry/Vermalem (1996) that fundamental value is a decision factor in deciding the contingent value of OMRs and Issuers' operative performance. Thus, the above result re-demonstrates that importance of getting the right fundamental value.

The test of joint repurchase performance has room for further improvements. For instance, it is unable to test whether practical OMRs trigger timing is consistent with capital investment theory and whether the trigger timing is consistent with the standard deviation of return or whether it is consistent with repurchase volume. To answer those theoretical arguments, more advanced analyses have to be analysed in the following section.

6.4.2 Theoretical hypothesis test to one-time repurchase model under a given repurchase volume

We intend to test hypothesis 1 in section 5.3. To conduct the test of optimality, this section employs the regression method to test the relationship between optimal trigger timing and the standard deviation of return; and to test the relationship between optimal trigger timing and repurchase volume. Same as the previous studies, both estimated fundamental values are employed to compute the practical repurchase timing. Therefore, there are four regression models which will be constructed and analysed. The analysed results are presented in Table 6-3.

[Insert Table6-3 about here]

Model 1 represents the regression models of endogenous variable, volatility of stock return, and exogenous variable, repurchasing timing under the first type of estimated fundamental value. Model 2 represents the regression models of endogenous variable, volatility of stock return, and exogenous variable, repurchasing timing under the second type of estimated fundamental value. Both models include small samples so we encounter the problem of pattern recognition.The analysed results show that both models have a positive relationship between the practical OMRs trigger timing and the standard deviation of return. We interpret that the joint practical OMRs operations are consistent with financial capital investment theory.[12] However, Model 1 and

[12] We have to note, however, that the results are not statistically significant, although the signs of the parameter estimates are all correct. This result can be attributed to the small sample size used and in future research an increased sample size would allow us to derive statistically significant results.

Model 2 also indicate that both models have poor explanatory abilities R^2 and poor effectiveness of model $(\Pr > F)$[13]. We explain that either part of sampled companies' trigger timings are not highly consistent with its/ their standard deviation of return(s) because a greater investment risk is expected to yield a greater contingent value. A poor explanatory ability and a poor effectiveness reveal that the sampled companies' repurchase timing is not highly consistent with capital investment theory. This analysis provides a deeper analysis to demonstrate the test of optimality. Based on the analysed results, we recommend that the sampled company has to adjust their repurchase timing with a positive trend of standard deviation of its stock return.

In the analysis of trigger timing and repurchase volume, Model 3 represents the regression models of endogenous variable given the repurchase volume and exogenous variable repurchasing timing under the first type of estimated fundamental value. Model 4 represents the regression models of endogenous variable given the repurchase volume and exogenous variable repurchasing timing under the second type of estimated fundamental value. Because both models include small samples the problem of pattern recognition occurs. Within both regression analyses, Model 4

[13] The p-value associated with this F value is used to answer "Do the independent variables reliably predict the dependent variable? The p-value is compared to your alpha level (typically 0.05) and, if smaller, we can conclude that the independent variables reliably predict the dependent variable". Conversely, if the p-value were greater than 0.05, you would say that the group of independent variables does not show a statistically significant relationship with the dependent variable, or that the group of independent variables does not reliably predict the dependent variable.

shows that there is a positive relationship between the theoretical optimal trigger timing and the given repurchase volume; but Model 3 is even worse as it shows a negative relationship between the theoretical optimal trigger timing and the given repurchase volume. Both models have poor explanatory abilities (low R^2) and poor effectiveness ($\Pr > F$)[14]. Most of the statistics in Model 3 and Model 4 shows that there are better repurchase volume strategies to be adopted.

Our demonstration can be supported by existing references by Ikenberry/Vermalen (1996) and Ikenberry *et al.*(2000). Reviewing the information of the repurchased volume in Table 6-1, the sampled company's repurchase volumes are lower than the United States and Canada which have higher repurchase volumes. Ikenberry/Vermalen (1996) and Ikenberry *et al.*(2000) even ignored the case of 2.5 % below repurchase volume but this is close to our biggest repurchase volume of sampled companies. Those apparently small repurchase volumes proved that they are not optimal repurchase volume whatever the proof from the demonstration of this study or the empirical case from other experienced countries.

[14] See as footnote 3 in this Chapter

6.4.3 Optimal trigger timing study- one-time repurchase with Optimal repurchase volume

Next, by employing the same analysis procedure, we calculate the optimal trigger timing with optimal trigger volume. This analysis aims to test practical operative performance as to whether it repurchases optimally in whatever repurchase timing and repurchase volume.

The calculated optimal trigger timing with the optimal trigger volume is represented in Table 6-2. Unfortunately, the calculated optimal trigger volume drifts away from the rational repurchase volume and the condition of repurchase volume, e.g., the repurchase volume condition which is 10 % below that of its issued shares.

Analysing the above unreasonable outputs, we can provide three explanations for this incomplete model development. First, the optimal repurchase volume does not consider the condition of threshold such as the optimal trigger timing which has to be greater than or equal to a certain value of λQ. There is also an unproven supposition about the threat of take-over. The model derivation of optimal trigger timing model with repurchase volume did not consider the threat of take-over, so that the result obtained was out of critical trigger timing. Second, the derivation of revenue function in *Chapter* 5 assumes that the repurchase volume has a linear positive influence on the price difference between fundamental value and market price; however this assumption does need a deeper discussion in the case of optimal trigger volume. Third, a repurchase programme is influenced by the entire investment environment such as a company's operating performance, company's repurchase strategy, market change, and public investors' participants. The developed optimal trigger timing with optimal repurchase volume model is developed only taking into consideration of mathematical optimization but ignoring the practical operational threat and the regulation of related rules, so that the obtained results are unreasonable. The optimal trigger timing with optimal repurchase volume model needs to be studied further with more thoughtful considerations.

Since the result of the optimal one-time repurchase model with optimal trigger volume is outside of reasonable trigger timing and trigger volume, we will not discuss it in the following comparison analysis on practical operations.

6.4.4 Optimal trigger timing study- sequential repurchases

1. **Estimation of Optimal trigger timing in sequential repurchases and the hypothesis to test the kink between first time buyback and the second time buyback**

We analyse our optimal sequential repurchase model by substitution of required input data. The analysed result of optimal sequential trigger timing is presented in Table 6-4. With respect to the sequential repurchase model in theoretical explanations, the first time buyback is the same as the one-time repurchase model because the instantaneous market price is the price dynamics without share repurchases and the expected return of the issuer is the same as that in the one-time repurchase model. The following stochastic diffusion is the change of price dynamics with share repurchases. In the following empirical investigation, we will address and investigate the differences of repurchase activity between optimal one-time repurchase model under a given repurchase volume and optimal sequential repurchase model.

[Insert Table 6-4 about here]

Incorporating the information in Tables 6-2 and 6-4, the first time optimal trigger value of the repurchase in a sequential model is roughly the same as the value of the one-time repurchase model. The optimal trigger value has a more noticeable difference between the first-time repurchase and the second-time repurchase for all sampled companies. However, the optimal trigger value has no big difference between the second-time repurchases and the following repurchases for all sampled companies. This result demonstrates that in hypothesis 2 there is a kink between the first time buyback and the second time buyback as discussed when analyzing the model in Ch 5.2.3. This phenomenon can be explained from a stochastic diffusion perspective: one is dynamic with share repurchase, while the other is dynamic without share repurchases. Before an issuer launches its repurchase activity, the stochastic diffusion is a dynamic without repurchase shock in the market. The kink is normal and is consistent with exceptional economic behaviour such as entry by a new competitor in a market with few firms, so that price suddenly drops. The new market price becomes a further fair market price.

The expected trigger value is the price difference at that time. Under the expectation of triggering an investment, an issuer will wait until the optimal timing to launch its investment to make a maximum profit, so that it has the biggest price difference at that time. When the first timing repurchase activity has been realised, the price difference is changed immediately because we assume that our repurchase strategy is able to salvage the undervalued market price. The stochastic diffusion of shares is a dynamic with a repurchase disturbance. The issuer still keeps the same expected return. The optimal trigger point is tangential to the differential curve which is the point on the stochastic diffusion curve with the previous repurchase influence, ie, $dg_s = -\lambda\overline{q} + \lambda\sigma_s dz$. Except for a kink between first time repurchase and second time repurchase, they are almost the same in the following repurchase timing/ repurchase volume.

2. Hypothesis test of joint sequential repurchase performance to save the undervalued prices

One important assumption of the optimal sequential repurchase model is that the price difference should decrease with increased execution of repurchase activities. This is exactly as with hypothesis 3 that repurchases are conducted to salvage the falling price. To test this hypothesis, we employ a regression analysis to test this hypothesis. The exogenous variable is the variable of the natural log current price divided by its lagged price, denoted as $\ln\frac{P_{t+1}}{P_t}$. This exogenous variable represents the rate of payoff of executing a stock repurchase. If this variable has a positive sign, the repurchase performance is consistent with hypothesis 3; otherwise, it is not. The endogenous variable is the given repurchase volume which is denoted as Δq_{t+1}. Because this model includes small samples the problem of pattern recognition occurs.

The analysed results are presented in Table 6-5; the result shows that there is an inverse relationship between repurchase volume and the rate of repurchase. This model also has a weak explanatory ability with respect to the input exogenous variable and endogenous variables. The inverse relationship between repurchase volume and the rate of payoff is consistent with the numerical analysis in *Chapter* 5 if practical OMRs are operated optimally. We have explained this phenomenon and will not re-interpret it again. Nevertheless, if practical OMRs did not operate optimally, we interpret this as a negative relationship between rates of payoff and repurchase volume. Under the price condition and volume condition, the ruled repurchase circumstance prevents market participants from manipulating the market price, so that the falling price did not rise. These repurchase conditions might result in a negative relationship between rate of payoff and repurchase volume.

As regards the poor explanatory ability in Model 5, similarly, this is likely due to the limitations under the price and volume conditions. These conditions are successfully preventing market participants from manipulating the market price, so that parts of repurchase activity effectively save the falling price but others do not. There is no clear trend between the endogenous variable and the exogenous variable.

6.4.5 Analysis of cumulative rate of return of executing OMRs

The numerical analysis in section 5.2 and the regression analysis of sequential repurchases in section 6.4.4 show that the interaction between repurchase volume and optimal trigger timing bear a negative relationship when repurchase volume is small. Though we have explained this phenomenon, people could still misunderstand that the small repurchase volume will discourage share repurchase. The numerical analyses in sections 5.2 and the regression analysis of sequential repurchases in section 6.4.4 did not demonstrate that the OMRs are able to salvage the undervalued stock price. In particular, the analysis in section 6.4.4 will provide a wrong impression that executing share repurchase could damage the investment return of repurchasing stock. The following analysis proves that employing cumulative returns in different time stages to explain that OMRs are an efficient tool of investment to salvage the undervalued price.

The analysis starts with collecting one year historical price data for 2004 and dividing it into three stages: ex-ante repurchase stage that is between January 1, 2004 and the day prior to the commencement of the share repurchase, the repurchase stage, and the ex-post stage which is after the expiry date of repurchase to the end of December 31, 2004. Since the sampled companies launched their repurchase programme on different days during 2004, each company has a different time length of ex ante period and ex post repurchase period. Therefore, the critical test index which will be employed is the average cumulative return, which is presented in Eq. (6.4)

$$ACR_j^i = \frac{\sum_{1}^{n_j^i} \ln \frac{P_{t+1}^i}{P_t^i}}{n_j^i} \qquad \text{(6-4)}$$

i: sampled company

j: time period

n_j^i: days of different company in different time period

[Insert Table 6-6 about here]

Table 6-6 shows that the average cumulative return (ACR) in the ex ante stage has a lower return than the repurchasing period for each sampled company whatever single company or joint means of cumulative return. The results presented here are fairly consistent with the results previously reported in the literature. This is because executing OMRs have successfully shrunk the magnitude of the falling price, so that executing OMRs are a good tool of investment. However, people will question why there is a negative relationship between single $\ln\frac{P_{t+1}}{P_t}$ and Δq but a positive contribution to the cumulative returns. This is because executing share repurchase has the ability to shrink the magnitude of falling price but it might be unable to reverse the falling pricing immediately [15]. This result is significant because it not only demonstrates that OMRs can be a tool of investment but it also solves the puzzle between the increased repurchase volume and the change of price difference between fundamental value and share price. Though this test method is simple, it makes for a seamless good analysis.

The joint cumulative return and joint average cumulative return of ex poste period is best amongst different time stages. We explain this phenomenon because the company increased investors' confidence by signaling the good health of the company's operation through buying back outstanding shares which the company viewed as

[15] The relative rules restrict the repurchase price such that this cannot be over the stock price of the previous transaction day in some countries.

undervalued, which in turn stimulated more investors to invest in the company. This result even re-enhances the function of share repurchase in OMR execution as a tool of investment. Based on this empirical investigation, we recommend that companies have to continue executing OMRs as a form of regular investment behaviour in order to save the undervalued price in a bear market.

6.5 Summary

The developed optimal trigger timing models provide a solution characterized by maximum profit. Therefore, it is suitable for launching OMRs for the purpose of maintaining a firm's credit standing and the rights of shareholders. Based on this principle, we sampled seven Taiwanese IT companies within the FTSE Taiwan 50 index which executed OMRs in 2004. These are UMC, Compal, TSMC, Acer, Realtek, Quanta and MTK.

The fundamental value is an important parameter to estimate the theoretical optimal trigger price and an important parameter to test practical OMRs operative performances. In this study, both theoretical and practical estimation methods are employed to calculate different fundamental values for further applications and to provide further explanations and comments.

For the theoretical optimal repurchase timing model study, both one-time repurchase model under a given repurchase volume, the one-time repurchase model with optimal repurchase volume, and the optimal sequential repurchase model are investigated. We employed the optimal one-time repurchase model under a given repurchase volume to test practical operations. We find that part of practical operations did not follow the capital investment theory to execute their OMRs programmes, such as those which did not execute their repurchase timing with consideration of the standard deviation of return. Besides, the test result shows that their repurchase volume strategies also have poor explanations for the optimal trigger timing study.

In addition, we also analyse the difference between the one-time repurchase model under a given repurchase volume and the optimal sequential model, though we did not find that there is a big difference in repurchase timing between the one-time model and the first time repurchase sequential model. However, there are kinks between the first time repurchase timing and the second-time repurchase timing amongst the sampled companies.

As OMRs have the extra function of saving the undervalued share price, we next conduct an analysis on the cumulative rate of return in different time stages: ex ante repurchase stage, repurchasing stage, and the ex-post repurchase stage. The empirical investigation demonstrates that executing share repurchase is able to save the falling market price.

6.6 Critical analysis

OMRs are regarded as a form of investment behaviour. Our developed models are able to derive the optimal trigger timing for such capital investment. This is a significant contribution in practice given that optimal trigger timing is important to investors.

We introduce in the first chapter the various powerful functions of OMRs such as adjustment of the capital structure, signaling private information, and takeover defense etc. Our developed models, however, cannot take all these functions into account due to the limiting problems of the modeling process and solution.

In addition, from conducting the above empirical study, we found that there are several input parameters which cannot be obtained easily such as the liquidity parameter, λ, and a company's fundamental value.

Furthermore, references also document that other investor considerations such as company size, dividend, reputation, and the experience of executing OMRs are also taken into account. We ignored such considerations but employ the market liquidity parameter, λ, instead. Unfortunately, the individual liquidity parameter cannot be obtained. We assume that each sample company has the same liquidity parameter in estimating each sampled company's optimal trigger timing though it is not very reasonable in practical OMRs operations.

Regarding the sample size for this empirical investigation, the sampled companies are the most representative companies in Taiwan, but the sample size is small. In the above empirical investigation, we are unable to avoid the error of small sample problem; the distribution of standard deviation does not follow normal distribution.

Chapter 7
Conclusions and further studies

7.1 Conclusions

OMRs have demonstrated their functions in terms of money distribution, employee compensation, adjustment of the capital structure, and as a takeover defense etc. An active function of executing OMRs is that it has been treated as a tool of investment. However, up-to-date, most of the references in this field aim to study the short-term share price change or the share return surrounding the announcement period, the starting day and the long term influence to the executing OMRs.

There is a lack of study providing optimal trigger timing to issuers. Aiming at this research gap, this study undertakes an optimal trigger timing study to OMRs under uncertainty. As OMRs are executed in open market, it could damage the benefit of public investors. Further, OMRs are characterised as a non-commitment money distribution strategy. To analyse its relative limitations and its inherent flexibilities is necessary before constructing an optimal trigger timing model to OMRs. When constructing a complete survey to the relative rules and regulations in several typical countries who allow executing OMRs, there are three important conditions that rule the execution of OMRs these are: price condition, volume condition, and timing/ duration condition. These conditions compose a safe harbour to protect public investors. Issuers of OMRs programme must have sufficient flexibilities within the above listed conditions to create maximum profit for their long-term shareholders.

In the aspect of investment, executing OMRs is a non-commitment sequential investment with limitations of investment price and volume conditions at each stage. Since OMRs has very similar characters with American type options, especially, OMRs is executed in an uncertain open market, by employing an option-based pricing method to construct an optimal trigger timing model and is naturally the best research method.

Our development process of construction begins with a one-time repurchase model under given repurchase volume, and then extends it the developed model to a one-time repurchase model with optimal repurchase volume. Finally, we extend the developed basic model to a sequential model. Similar with the well-known Black-Scholes model, our developed one-time repurchase model requires five parameters which are: risk free rate, total issued shares, total repurchase volume (this is not needed for the one-time repurchase model with optimal repurchase volume), liquidity parameter, and standard deviation of return. The developed sequential model requires the above five parameters plus an additional three which are: maximum talent trigger value, repurchase frequency, and average repurchase volume at each stage. These three developed models are able to provide the best trigger timing of OMRs.

After the developed models were completed, a numerical analyses to verify the effectiveness of developed models was undertaken. The numerical analysis of two of our one-time repurchase volume models (i.e. the optimal trigger timing model with given repurchase volume and the optimal trigger timing model with optimal repurchase volume) confirms they are consistent with capital investment theory. They have both a positive relationship between the variance and the trigger timing and both have a negative relationship between risk free rate and trigger timing. In other words, the numerical analysis points that a higher repurchase risk combines a bigger trigger timing to yield a bigger profit, and a higher risk free rate pushes issuers to trigger repurchase activity earlier. Both results demonstrate our one-time repurchase models are consistent with capital investment theory. Besides, the sequential repurchase model, the numerical test results are mostly the same as to the two one-time repurchase models except that there is a negative relationship with optimal trigger timing in the condition of small repurchase volume. We can explain this finding by considering the profits the company would make from the repurchase as defined by its objective function. When repurchasing a small amount, the profits will be relatively small. Thus, when the values differ only marginally, it is beneficial to wait for a larger difference as this would translate to a higher profit.

Finally, an empirical examination to test the application of practical operation was conducted. The sampled companies comprise in part the FTSE Taiwan 50. These are: Acer, Compal, MTK, Quanta, Realtek, TSMC, and UMC. The fundamental value of each company is important because it reveals whether or not to launch a OMRs and where the optimal trigger timing point occurs. The complete empirical investigation is summarised as follows:

Optimal one-time repurchase timing model under given repurchase volume:

1. The theoretical optimal trigger timings are between 0.541~0.676. In other words, when market price falls down to $\frac{1}{1..97} \sim \frac{1}{1.72}$ market price issuers should execute a buyback programme to create maximum profit for their long-term shareholders.
2. Using the fundamental value derived from financial theory, the test of joint repurchase performance reveals that the sampled companies had triggered their OMRs too late, so that their market prices are undervalued for too long a time. They potentially exposed their companies to the threat of take-over.
3. Using the fundamental value derived from finance practice in Taiwan, we see that the test of joint repurchase performance reveals that the sampled companies repurchase optimally.
4. With both estimated fundamental values, the theoretical test of higher risk combining with a bigger trigger volume points than the sampled companies have a positive response to this theory. However, the weak explanation and poor model effectiveness also prove that part of our sample companies did not trigger OMRs at the optimal trigger timing.
5. With estimated theoretical fundamental values, the theoretical test of a bigger repurchase volume combined with a bigger repurchase timing points that there is a negative relationship between repurchase volume and repurchase timing. This

result violates capital investment theory. With estimated practical fundamental values, the theoretical test of a bigger repurchase volume combine with a bigger repurchase timing points that there is positive relationship between repurchase volume and repurchase timing. This result is consistent with capital investment theory. However, a poor model explanation and a poor model of effectiveness revealed that the sampled companies remain to improve their repurchase volume strategy.

Optimal one-time repurchase timing with given repurchase volume model:

The developed optimal one-time repurchase timing with given repurchase volume model fails to provide a realistic repurchase strategy. The lacking considerations of repurchase volume restriction and price-volume behaviour in the solution process is with the result that optimal trigger volume is out of the rules' regulation.

The model of optimal sequential repurchase timing under given repurchase volume:

1. The estimated sequential optimal trigger timing result reveals there is a kink between the first time repurchases and the second time repurchases, but then the following trigger timings remain stable. The kink is consistent with rational economic behaviour. It thus demonstrates the validity of the developed optimal

sequential repurchase timing model.

2. The analysis to the relationship between repurchase return and repurchase volume reveals that there is a negative relationship between both of them. However, this result suposes that it is influenced by ineffective market response and relative repurchase price restriction. To provide a deeper analysis to prove OMRs can be a tool to salvage undervalued market price, a further analysis of average cumulative return is studied.
3. The cumulative return analysis is designed to divide time as three stages: ex ante repurchase stage, repurchasing stage, and ex post repurchase stage. The statistic tests to those three stages are significant in that the average cumulative return are different in these three stage. In addition, the analysed result reveals that the average cumulative return of ex post stage is better than repurchasing stage; also the average cumulative return of repurchasing stage is better than ex ante stage. These results demonstrate the power function of executing OMRs can be used to salvage the undervalued market price.

7.2 Further studies

Further study can move forward in the directions: one is study the required input variables for the estimation and verification of model. Another approach to further the study is to extend the model to include more realistic features such as including the time value of the option.

Regarding input variables, such as market liquidity, an individual company has different characteristics such as firm size and reputation etc. which yield different market liquidity when a company announces a buyback programme. Alas, to assure all sampled companies have same market liquidity is unrealistic. In addition, as mentioned in previous references, the fundamental value is an important parameter to decide whether implement OMRs and to decide the option value/ optimal trigger timing. Although, there are two different fundamental value estimation methods are provided, but part estimated fundamental values are far-fetch to the reality. Creating an objective and accurate estimation method is thus important.

In the aspect of model development, optimal one-time repurchase timing with given repurchase volume model fails to provide realizable repurchase strategy. The further study can start from this drawback to begin improvements such as price- volume behaviour of shares under repurchase announcement. Also the enhancing optimal sequential repurchase timing with given repurchase volume model can be extended to sequential optimal repurchase timing and the optimal repurchase volume model to provide a closer realistic assistant decision-making technology.

Although the OMRs signalling effect is hard to appraise, the signal does positively effect a company's effort to salvage an undervalued share price, McNally (1999) and Isagawa (2000, 2002). It is worthwhile to include the signalling effect into the optimal trigger timing model .

Finally, a game choice option-based model also merits further study. This model must take into account both the price-volume behaviour and the market response from other market participants for the model to be complete. This requires more information between issuers and other market participants.

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Table 2-2 the relative rules of OMRs of sampled countries

Condition / Country	Approval	Timing Restriction	Price Restriction	Volume Restriction
USA	None	No repurchase during last 30 mins of a trading day	No higher than highest current price or last independent sale price	# (in September 2001 restrictions on daily repurchase volume were suspended)
Japan	No need after 1997	No repurchase during last 30 mins of a trading day	No higher than last day price	No more higher than 25% trading day
UK	Shareholder meeting	18 mo limit	No higher than 5% of 5 day average price	15% of total shares
France	Approval by CBO	18 mo limit, 15 days before EA	No higher than daily high	10% of total shares, 25% of daily volume
Germany	Shareholder meeting	18 mo limit	Shareholder meeting: max and min have to be determined	10% of total shares
Canada	Board	12 mo limit	No higher than highest current price or last independent sale price	No more 10% of public float, 5% of total shares, or 2% in 20 days

Table 2-2 the relative rules of OMRs of sampled countries (continuous)

Condition / Country	Approval	Timing Restriction	Price Restriction	Volume Restriction
Italy	Shareholder meeting	18 mo limit	No higher than most recent price	10% of total shares, 25% of monthly volume
Netherlands	Shareholder meeting	18 mo limit	Shareholder meeting: max and min	10% of total shares
Switzerland	Board	#	#	10% of total shares
Hong Kong	Shareholder meeting	12 mo limit; and 1 mo before earning announcement	#	10% of total shares, 25% of monthly volume
Taiwan	Board	12 mo limit	No higher than last day price	10% of total shares, 25% of average turnover

Source: Kim *et al.* (2003) and collection of this study

Remark: # represents no record about the regulation

Table 6-2 Optimal trigger timings of one-time repurchase models

The following table summaries the relative variables to estimate the optimal one-time repurchase models and the optimal trigger timings, given repurchase volume and optimal repurchase volume. Since there are two different estimated fundamental value, the joint optimality tests for one-time repurchase model with given repurchase volume are two. The joint optimality tests for one-time repurchase model with optimal repurchase volume are ignored because the unrealized estimated trigger timings. (Critical value of t-test for sample size 4 is 2.571; Critical value of t-test for sample size 5 is 2.447)

Var \ Com	Acer	Compal	MTK	Realtek	Quanta	TSMC	UMC
g^*	0.541560613	0.5955079	0.59225904	0.676282204	0.583439388	0.562504507	0.593614049
g^{**}	1.071160481	1.179801315	1.177599487	1.345261161	1.160344456	1.11813332	1.178546088
Q^{**}	4.214844107	4.642328303	4.633664466	5.293386169	4.565768693	4.399674669	4.637389187
g_1	0.409030174	--	0.357013365	0.543474572	--	0.385124832	0.47210244
g_2	--	0.767514955	0.46226722	0.775235777	0.626785257	0.474381529	0.780063603
$t_{(g^*/g_1)}$	-3.497305964						
$t_{(g^*/g_2)}$	1.236415033						

g^* : optimal trigger timing with given repurchase volume; g^{**} : optimal trigger timing with optimal repurchase volume; Q^{**} : optimal trigger volume; g_1 : practical trigger timing with theoretical fundamental volume; g_2 : practical trigger timing with theoretical fundamental volume

Table 6-3 Theoretical hypothesis test

Model 1 and Model 2 demonstrate the relationship between practical trigger timing and the standard deviation of return. Model 3 and Model 4 demonstrate the relationship between practical trigger timing and the repurchase volume. Between Model 1 and Model 3, the sample companies of Compal and Quanta are ignored because they have unrealistic trigger timing that is estimated with practical fundamental evaluation. Hence, there are five sampled companies in the studies. Similarly, Acer is ignored in Model 2 and Model 4.

Hypothesis 1: the optimal trigger timing has a positive relationship with the standard deviation of return, σ_s ***and the optimal trigger timing has a positive relationship with the repurchase volume,*** Q					
	Parameter Estimate		**t Value**	**R-Square**	**Pr > F**
Model 1	Intercept	-2.22341	-0.71	0.5969	0.1256
	Stand derivation of return	3.47021	2.11		
	$g_1 = a_1 + b_1\sigma_s$				
Model 2	Intercept	-0.62248	-0.65	0.3056	0.2553
	Stand derivation of return	6.62632	1.33		
	$g_2 = a_2 + b_2\sigma_s$				
Model 3	Intercept	0.44631	7.92	0.0313	0.7758
	Q	-1.36467	-0.31		
	$g_1 = a_3 + b_3 Q$				
Model 4	Intercept	0.25346	6.24	0.2234	0.3439
	Q	9.88056	1.07		
	$g_2 = a_4 + b_4 Q$				

Table 6-4 Optimal trigger timing of sequential repurchases

This optimal sequential repurchase timing analysis is conducted by substituting each company's different repurchase frequency, given repurchase volume and the other necessary input variables into the model to calculate the optimal sequential repurchase timing. Therefore, every repurchase has optimal repurchase timing. i.e. TSMC has 14 optimal repurchase timing consistent with its 14 repurchase activities. The comparison to the trigger timings between first time buyback and the second time buyback demonstrates there is a kink between two of them. In other words, the hypothesis is proved in this analysis.

Repur. Times	Acer	Compel	MTK	Quanta	Realtek	TSMC	UMC
1	0.66547	0.58977	0.58864	0.57974	0.67254	0.55904	0.58927
2	0.52792	0.48800	0.58127		0.38797	0.54548	0.54003
3	0.52794	0.48803				0.54548	0.54003
4	0.52793	0.48802				0.54548	0.54001
5	0.52780	0.48801				0.54547	0.54001
6	0.52793	0.48801				0.54546	0.53999
7	0.52793	0.48791				0.54547	0.54000
8						0.54547	0.54000
9						0.54547	
10						0.54547	
11						0.54548	
12						0.54547	
13						0.54547	
14						0.54547	

Table 6-5 Theoretical hypothesis test

Model 5 intends to demonstrate whether a practical repurchase activity is able to salvage the undervalued stock price. In this analysis, we initially collect the repurchase return and the repurchase volume of every repurchase activity of each sampled company followed by a regression analysis to analyse the relationship between the endogenous and exogenous variables. [16]

Hypothesis 3: the repurchase activity is able to save the undervalued share price					
	Parameter Estimate		**t Value**	**R-Square**	**Pr > F**
Model 5	Intercept	-0.00172	-0.16	0.0115	0.5468
	Δq_i	-3.01479	-0.61		
	$\ln\frac{P_{i+1}}{P_i} = a_5 + b_5 \Delta q_i$				

[16] The repurchase shock is defined as $\lambda q / \lambda Q$. In this analysis, λ is ignored because λ is assumed to be the same for all sampled companies

This sketch figure shows the different time periods in 2004 - ex ante period, repurchasing period, and ex-poste period. The time length of the repurchase period is two months for each sampled company but the time lengths of the ex ante period and ex post period of each company are different because sampled companies executed OMRs on different repurchase days. The cumulative return is calculated by taking $\sum \ln \frac{P_{t+1}}{P_t}$ at different time periods.

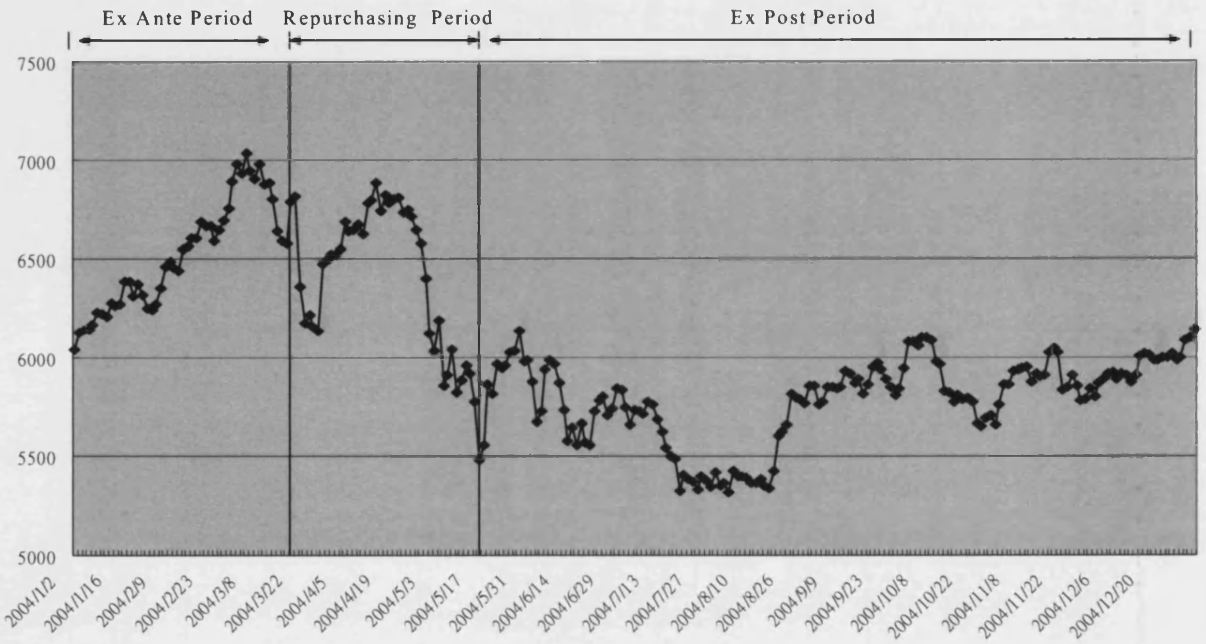

Figure 6-3: Different time periods in 2004

Table 6-6 Test of cumulative return to each sample company

	Ex Ante Stage		Repurchasing Stage		Ex Poste Stage	
Return	C.R.	A.C.R.	C.R.	A.C.R.	C.R.	A.C.R.
Acer	-0.0518	-0.001	-0.0396	-0.0009	0.2307	0.0015
Compal	-0.1157	-0.0021	-0.0337	-0.0008	-0.1177	-0.0008
MTK	-0.1201	-0.0014	-0.0977	-0.0023	0.5683	0.0021
Quanta	-0.2353	-0.0029	0.0445	0.0011	-0.0509	-0.0004
Realtek	-0.3418	-0.0042	0.0339	0.0008	-0.0091	-0.0001
TSMC	-0.1747	-0.0034	-0.1747	-0.0019	0.0677	0.0004
UMC	-0.0726	-0.0014	-0.0144	-0.0003	-0.2131	-0.0014
Mean	-0.15886	-0.00234	-0.04024	-0.00061	0.06799	0.00019
SD	0.101513	0.001191	0.076125	0.001268	0.26152	0.001248
$t_{ex\ ante \rightarrow ex\ post}$	-15.3502 (p=-0.0000)					
$t_{ex\ ante \rightarrow repurchasing}$	-8.54506 (p=-0.0000)					
$t_{repurchasing \rightarrow ex\ ante}$	-5.4365 (p=-0.0016)					

Remark: C.R.: cumulative return; A.C.R.: average cumulative return; S.D.: Standard deviation of return

Appendix A: Derivation of Differential Equation/ Diffusion Equation

A contingent value is considering as a function that it is twice differential in S and once in t, denote it as $F(S,t)$. Among this derivation, S is assumed that the underlying asset and it's instantaneously change is a simple Brown-Wiener process:

$$dF = \frac{\partial F}{\partial S} dS + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS)^2 + \frac{\partial F}{\partial t} \tag{1}$$

$$d(\ln S) \approx \frac{dS}{S} = \alpha_S dt + \sigma_S dz \tag{2}$$

Eq.(4-2) have an optimal stopping problem in continuous time. $F(S,t)$, yields no cash flows up to the time of exercise (T) that the investment is undertaken, the only return from holding it is its capital appreciation. Hence in the continuous region, the above exercise strategy can be written as Eq. (3)

$$\rho F dt = E(dF) \tag{3}$$

For simplicity, ignoring the influence of time factor to the contingent value, then dF is expanded by ordinary derivation as follows:

$$
\begin{aligned}
E(dF) &= E\left(F_t dt + F_S dS + \frac{1}{2} F_{SS} (dS)^2 \right) \\
&= E\left(F_t dt + F_S (\alpha_S S dt + \sigma_S S dz) + \frac{1}{2} F_{Ss} (\alpha_S S dt + \sigma_S S dz)^2 \right) \\
&= E\left(F_t dt + \alpha_S S F_s dt + \frac{1}{2} \sigma_s^2 S^2 F_{ss} dt \right)
\end{aligned}
\tag{4}
$$

Incorporating Eq (3) into Eq. (4), then the optimal differential equation can be obtained as Eq. (5)

$$
\frac{1}{2}\sigma_s^2 S^2 F_{SS} + \alpha_S S F_S - \rho F = -F_t \tag{5}
$$

In a real world investment consideration, the instantaneous change of the option value in time factor could be vanishing at two scenarios. One is that the influence factor has only a slight influence on the option value. People can simply ignore its existence. The other scenario is that a real investment can last for a longer project life. In this case, Eq. (4-4) can be derived as

$$
\frac{1}{2}\sigma_s^2 S^2 F_{SS} + \alpha_S S F_S - \rho F = 0 \tag{6}
$$

2. Solution Process:

Eq. (6) is subject to the following constraints

$$\lim_{S \to 0} F(S,t) = 0 \tag{7}$$

$$F(\mathrm{S},t) = S - C \tag{8}$$

$$F_S(S,t) = 1 \tag{9}$$

Eq. (6) is an ordinary Eular equation. It has the general solution of

$$F = A_1 S^{\beta_1} + A_2 S^{\beta_2} \tag{10}$$

To solve β, we can write the general form as $F = AS^{\beta}$ for simplicity.

Substituting the general solution into Eq. (6), then

$$\frac{1}{2}\sigma_s^2 S^2 \left(A\beta(\beta-1)S^{\beta-2}\right) + \alpha_S S\beta A S^{\beta-1} - \rho A S^{\beta} = 0$$

and

$$\beta = \frac{\left(-\alpha_S + \frac{1}{2}\sigma_s^2\right) \pm \sqrt{\left(\alpha_S - \frac{1}{2}\sigma_s^2\right)^2 + 2\rho\sigma_s^2}}{\sigma_s^2} \tag{11}$$

$$\beta_1 = \frac{\left(-\alpha_S + \frac{1}{2}\sigma_s^2\right) + \sqrt{\left(\alpha_S - \frac{1}{2}\sigma_s^2\right)^2 + 2\rho\sigma_s^2}}{\sigma_s^2}$$

and

$$\beta_2 = \frac{\left(-\alpha_S + \frac{1}{2}\sigma_s^2\right) - \sqrt{\left(\alpha_S - \frac{1}{2}\sigma_s^2\right)^2 + 2\rho\sigma_s^2}}{\sigma_s^2}$$

The constraint of Eq. (7) enforces A_2 to be equal to zero. Therefore, the general solution of Eq (6) remains as

$$F = A_1 S^{\beta_1} \tag{12}$$

Next, we can depend on the general solution of Eq. (8) with the value matching function and the high contact conditions to solve the optimal trigger point.

Firstly, differentiating Eq. (12) by S and then equating the new differential equation to Eq. (9), we can solve it as

$$F_S = A\beta\left(S^*\right)^{\beta-1} = 1$$

or

we can write $A=\frac{1}{\beta\left(S^{*}\right)^{\beta-1}}=\frac{\left(S^{*}\right)^{1-\beta}}{\beta}$ (13)

Substituting Eq. (13) into Eq. (12) and putting it to be equal to Eq. (8),

then, we can solve the optimal contingent value evaluation model with boundary conditions

$$\frac{\left(S^{*}\right)^{1-\beta}}{\beta}\times S^{*}=S^{*}-C$$

$$S^{*}=\frac{\beta_1}{\beta_1-1}C \tag{14}$$

$$F\left(S^{*},t\right)=\frac{(\beta_1-1)^{\beta_1-1}}{(\beta_1)^{\beta_1}C^{\beta_1-1}}\times\left(\frac{\beta_1}{\beta_1-1}\right)^{\beta_1}C^{\beta_1}=\frac{C}{\beta_1-1} \tag{15}$$

and the coefficient of the general solution is

$$A_1=\frac{S^{*}-C}{\left(S^{*}\right)^{\beta_1}}=\frac{(\beta_1-1)^{\beta_1-1}}{(\beta_1)^{\beta_1}C^{\beta_1-1}} \tag{16}$$

Thus, we obtain the optimal S^{*} and F^{*}.

Appendix B: Ordinary Differential Equation Derivation: One-time Repurchases

Bellman's Principle of Optimality which indicates the total expected return on the investment opportunity is equal to its expected rate of capital appreciation that over a time interval dT. Mathematically, it can be represents as

$$\rho F dT = E(dF) \tag{1}$$

Taking approximation by Taylor expansion to the right-hand-side of Eq. (1) then we can have

$$dF = F_t dt + F_g(g)dg + \frac{1}{2}F_{gg}(g)dg^2 \tag{2}$$

Under the expectation theory, the Eq. (2) can be rewritten as follows

$$E[dF] = E\left[F_t dt + F_g(g)\lambda\sigma_s dz + \frac{1}{2}F_{gg}(g)\lambda^2\sigma_s^2 dt\right]$$

Since the expected dz is equal to zero, and we assume time factor just has small influence to the contingent value change[17], therefore

[17] The life of one-time repurchase model or a real investment is long so that the time factor has slightly effect to the contingent value change. We disregard this influence to the contingent value

$$E[dF] = E\left\{F_g(g)[(-\lambda q)dt] + \frac{1}{2}F_{gg}(g)(\lambda^2\sigma_s^2 dt)\right\} \tag{3}$$

Substituting Eq. (3) into Eq. (1), then we can have

$$\frac{1}{2}F_{gg}(g)\left(\lambda^2\sigma_s^2\right) - \delta F = 0 \tag{4}$$

Eq. (4) subjects to the following conditions

$$\lim_{g \to 0} F(g) = 0 \tag{5}$$

$$F(g) = QN\left(g - \frac{1}{2}\lambda Q\right) \tag{6}$$

$$F_g(g) = QN \tag{7}$$

Eq. (4) is the standard ordinary differential equation which has the characteristic of solution non-repeated to zero. Its general solution is exponential type, write it as

$$F(g) = A_1 e^{\beta_1 g} + A_2 e^{\beta_2 g} \tag{8}$$

The Eq. (5) rules out the second part of general solution, Eq.(8). Therefore the final general solution of Eq. (4) remains as

$$F(g) = A_1 e^{\beta_1 g} \tag{9}$$

Differentiating Eq. (9) and ask it is equal to Eq. (7)

$$F(g) = A_1 \beta_1 e^{\beta_1 g^*} = QN$$ [18]

or

[18] The sub-notation represents tangent point or optimal trigger point

$$A_1 = \frac{QN}{\beta_1} e^{-\beta_1 g^*} \tag{10}$$

Substituting Eq. (10) into Eq. (9) and ask it is equal to Eq. (6)

$$\frac{QN}{\beta_1} e^{-\beta_1 g^*} e^{\beta_1 g^*} = QN\left(g^* - \frac{1}{2}\lambda Q\right)$$

$$g^* = \frac{1}{\beta} + \frac{1}{2}\lambda Q \tag{11}$$

Substituting Eq. (11) into Eq. (6), then

$$F\left(g^*\right) = \frac{QN}{\beta} \tag{12}$$

Substituting Eq. (11) into Eq. (6), then

$$A_1 = \frac{QN}{\beta_1} e^{-\beta_1 g^*} = \frac{QN}{\beta_1} e^{-\beta_1\left(\frac{1}{\beta}+\frac{1}{2}\lambda Q\right)} = \frac{QN}{\beta_1} e^{-\frac{1}{2}\lambda Q\beta_1 - 1} \tag{13}$$

Substituting Eq. (13) into Eq. (9), then

$$F(g) = A_1 e^{\beta_1 g} = \frac{QN}{\beta_1} e^{-\beta_1 g^*} e^{\beta_1 g} = \frac{QN}{\beta_1} e^{\beta_1\left(g-g^*\right)} = \frac{QN}{\beta_1} e^{\beta_1\left(g-\frac{1}{\beta}-\frac{1}{2}\lambda Q\right)} = \frac{QN}{\beta_1} e^{\beta_1\left(g-\frac{1}{2}\lambda Q\right)-1} \tag{14}$$

The complete solution process is completed.

Appendix C: Mathematical proof to the developed models follow capital investment theory

Eq. (5-13) $\beta = \frac{\sqrt{2\rho}}{\lambda\sigma_s}$, and Eq. (5-14) $g^* = \frac{1}{\beta} + \frac{1}{2}\lambda Q$ reveals the results of the one-time repurchase model under given repurchase volume. Substituting Eq. (5-13) into Eq. (5-14), we then can rewrite Eq. (5-14) as

$$g^* = \frac{\lambda\sigma_s}{\sqrt{2\rho}} + \frac{1}{2}\lambda Q \qquad (1)$$

If the σ_s is a variable, we can differentiate g^* by σ_s, which becomes

$$\frac{dg^*}{d\sigma_s} = \frac{\lambda}{\sqrt{2\rho}} \qquad (2)$$

Since λ and ρ are positive, we prove that the optimal trigger timing of the one-time repurchases under given repurchase volume has a positive relationship with σ_s.

Similarly, the optimal trigger timing derived from the model of the optimal trigger timing with optimal repurchase volume is equal to $g^{**} = \frac{2}{\beta}$. Substituting Eq. (5-13) into g^{**}, we can then have

$$g^{**} = \frac{2\lambda\sigma_s}{\sqrt{2\rho}} \tag{3}$$

If the σ_s is a variable, differentiating g^* by σ_s, then becomes

$$\frac{dg^{**}}{d\sigma_s} = \lambda\sqrt{\frac{2}{\rho}} \tag{4}$$

Since λ and ρ are positive, we prove that the optimal trigger timing of the one-time repurchases with optimal repurchase volume has a positive relationship with σ_s.

Finally, if the given repurchase volume is changeable, differentiating Eq. (5-14) by Q, then becomes

$$\frac{dg^*}{dQ} = \frac{\lambda}{2} \tag{5}$$

Eq. (5) also demonstrates that the optimal trigger timing under given repurchase volume has a positive relationship with Q.

Appendix D: Partial Differential Equation Derivation - Sequential Repurchases

The contingent function is a real function with two real variables. The differentiation of contingent function is sum up the partial differential equations. In a discrete representation, it can be presented as follows:

$$F(g+\Delta g,k+\Delta k)=F(g,k)+\left[F_g(g,k)\Delta g+F_k(g,k)\Delta k\right] \\ +\frac{1}{2!}\left[F_{gg}(g,k)(\Delta g)^2+2\Delta g\Delta kF_{gk}(g,k)+F_{kk}(g,k)(\Delta k)^2\right] \quad (1)$$

or

$$\Delta F(g,k)=\left[F_g(g,k)\Delta g+F_k(g,k)\Delta k\right] \\ +\frac{1}{2!}\left[F_{gg}(g,k)(\Delta g)^2+2\Delta g\Delta kF_{gk}(g,k)+F_{kk}(g,k)(\Delta k)^2\right] \quad (2)$$

when Δ is very small, we can represent the discrete equation to be a continuous type

$$dF(g,k)=\left[F_g(g,k)dg+F_k(g,k)dk\right] \\ +\frac{1}{2!}\left[F_{gg}(g,k)(dg)^2+2F_{gk}(g,k)dgdk+F_{kk}(g,k)(dk)^2\right] \quad (3)$$

$$dg=-\lambda\bar{q}dt+\lambda\sigma_s dz \quad \text{and} \quad dk=-\bar{q}dt \quad (4)$$

Substituting Eq. (4) into Eq. (3) with expectation

$$E\left(dF(g,k)\right)=E\left\{\begin{array}{l}\left[F_g(g,k)\left(-\lambda\bar{q}dt+\lambda\sigma_s dz\right)+F_k(g,k)\left(-\bar{q}dt\right)\right]\\ +\dfrac{1}{2!}\left[\begin{array}{l}F_{gg}(g,k)\left(-\lambda\bar{q}dt+\lambda\sigma_s dz\right)^2\\ +2F_{gk}(g,k)\left(-\lambda\bar{q}dt+\lambda\sigma_s dz\right)\left(-\bar{q}dt\right)\\ +F_{kk}(g,k)\left(-\bar{q}dt\right)^2\end{array}\right]\end{array}\right\} \quad (5)$$

The expected dz is equal to zero and the value of $dtdz$ and $(dt)^2$ are too negligible. We neglect the value of $dtdz$ and $(dt)^2$. Therefore, Eq. (5) can be rewritten as Eq. (6).

$$E\left(dF(g,k)\right)=\frac{1}{2}F_{gg}\lambda^2\sigma_s^2(g,k)dt-\lambda\bar{q}F_g(g,k)dt-qF_k(g,k)dt \quad (6)$$

Since Bellman tells $\rho F(g,k)dt=E\left(dF(g,k)\right)$ (7)

Therefore $\frac{1}{2}F_{gg}\lambda^2\sigma_s^2(g,k)-\lambda\bar{q}F_g(g,k)-\rho F(g,k)-qF_k(g,k)=0$ (8)

Appendix E: Model Derivation and Explanation - Sequential Repurchases

$$\rho F(g,k)\,dt = E\left(dF(g,k)\right) \qquad (1)$$

Eq. (1) is the Bellman's Principle of Optimality which indicates that over a time interval dt, the total expected return on the investment opportunity, $\rho F dt$, is equal to its expected rate of capital appreciation. Then $dF(g,k)$ is expanded by ordinary derivation, and the optimal contingent value function can be presented as follows:

$$\frac{1}{2}F_{gg}\lambda^2\sigma_s^2(g,k) - \lambda\bar{q}F_g(g,k) - \rho F(g,k) - qF_k(g,k) = 0 \qquad (2)$$

Eq. (2) is a partial differential equation that considers the influence of asymmetry payoff and the total remaining expenditure to the contingent value of OMRs. Eq. (2) has many solutions, corresponding to all the different derivations that can be defined with g and k as the underlying variables. The particular derivative obtained when the equation is solved depends on the boundary conditions that are used. These specify the values of the derivative at the boundaries of possible values of g and k. In the case of stock repurchases, the key boundary condition that we can observe is the limitations of asymmetry payoff, g. The boundary conditions that are imposed on g are:

$$\lim_{g \to -\infty} F(g,k) = 0 \text{ and} \tag{3}$$

$$\lim_{g \to \bar{g}} F(\bar{g},k) = B(\bar{g}) \tag{4}$$

Eq. (3) is the result of the fact that if the market share price tends to an extreme small value or the fundamental value ever reaches zero, then the repurchase programme will never be exercised and will become worthless. Eq. (4) is the result of the fact that if the market share price has fallen to a certain price, then the Issuer will face an immediate threat of take-over. $\bar{g}$ is the critical price before this threat appears and $\bar{g}$ is correspondent with a contingent value $F(\bar{g},k) = B(\bar{g})$. $\bar{g}$ is given by the professional Issuers' judgment. Therefore $\lim_{g \to \bar{g}} F(\bar{g},k) = B(\bar{g})$ is a constant. Mathematically, Eq. (3) and Eq. (4) are boundary conditions of Eq. (2). Besides, the solution of Eq (2) requires an initial solution, which is when $k = 0$. In order to cover all potential solutions of Eq.(2), the value of the option at the initial stage, $F(g,0)$, is assigned to be equal to $h(g)$, as indicated in Eq. (5).

$$F(g,0) = h(g) \tag{5}$$

With the above proper boundary conditions, then Separable Differential Equations can be employed to solve Eq. (2). The solution technique - variable separation, separates the whole result into two parts: a steady state result and a transit state result. The part of steady state represents the influence of variable g on the contingent sequential repurchase value; the other part of the transit result represents the influence of variable k on the contingent sequential repurchase value. Eqs. (3), (4) and (5) are sufficient and are necessary conditions for the general solution of Eq. (2) to be obtained.

Solution Process

First, let $F(g,k)=\phi(g,k)+w(g)$, and therefore Eq. (2) can be rewritten as

$$\frac{1}{2}\lambda^2\sigma_s^{\,2}(\phi_{gg}+w_{gg})-\lambda q(\phi_g+w_g)-\rho(\phi+w)=q\phi_k \tag{6}$$

Eq. (6) is consistent with Eq.(2) and it can be separated as Eq. (7) and Eq. (8)

$$\frac{1}{2}\lambda^2\sigma_s^{\,2}\phi_{gg}-\lambda q\phi_g-\rho\phi=q\phi_k \tag{7}$$

$$\frac{1}{2}\lambda^2\sigma_s^{\,2}w_{gg}-\lambda qw_g-\rho w=0 \tag{8}$$

Since the original partial differential equation, Eq. (2), has been transformed as two functions of $\phi(g,k)$ and $w(g)$, the boundary conditions, Eqs. (3) and (4), also need to be transformed. The new boundary conditions are shown as follows:

$$\lim_{g\to-\infty} F(g,k) = \lim_{g\to-\infty} \phi(g,k) + \lim_{g\to-\infty} w(g) = 0$$

$$F(\overline{g},k) = \phi(\overline{g},k) + w(\overline{g}) = B(\overline{g})$$

or

$$\lim_{g\to-\infty} \phi(g,k) = 0, \phi(\overline{g},k) = 0 \tag{9}$$

$$\lim_{g\to-\infty} w(g) = 0, w(\overline{g}) = B(\overline{g}) \tag{10}$$

Eq. (8) is an ordinary differential equation and its result can be written as

$$w(g) = B(\overline{g})e^{-m_1\overline{g}}e^{m_1 g} = B(\overline{g})e^{m_1(g-\overline{g})} \tag{8a}$$

$$m_1 = \frac{q+\sqrt{q^2+2\sigma_s{}^2\rho}}{\lambda\sigma_s{}^2} \tag{8b}$$

Regarding the solution process of Eq. (7), one needs to employ the Separable Differential Equation again

$$\text{Let } \phi(g,k) = G(g)K(k) \tag{11}$$

$$\frac{1}{2}\lambda^2\sigma_s{}^2\frac{G''}{G} - \lambda q\frac{G'}{G} - \rho = q\frac{K'}{K} = \alpha \tag{12}$$

or

$$\frac{1}{2}\lambda^2\sigma_s^{\ 2}G'' - \lambda qG' - (\rho+\alpha)G = 0 \tag{13}$$

$$K' - \frac{\alpha}{q}K = 0 \tag{14}$$

Eq.(12) is an ordinary differential equation, hence by ordinary solution process, and by letting $G(g) = A \cdot e^{ng}$, we obtained the result below as follows:

$$n_{1,2} = \frac{q \pm \sqrt{\mu}}{\lambda\sigma_s^{\ 2}} \tag{15}$$

where $\mu = q^2 + 2\sigma_s^{\ 2}(\rho+\alpha)$ is an eigenvalue

By letting $\mu = p^2$ to facilitate the following discussion,

There are three cases to consider for the constant, μ, in Eqs. (13) and (14): whether it is positive, zero, or negative

when μ is negative, the non-trivial solution can be obtained, and the eigenfunction is

$$\therefore G(g) = ce^{\frac{qg}{\lambda\sigma_s^{\ 2}}}\left[-\tan\frac{p\bar{g}}{\lambda\sigma_s^{\ 2}}\cos\frac{pg}{\lambda\sigma_s^{\ 2}} + \sin\frac{pg}{\lambda\sigma_s^{\ 2}}\right] \tag{16}$$

Eigenvalue： $\mu = -p^2 = q^2 + 2\sigma_s^{\ 2}(\rho+\alpha) < 0$

$$\alpha < \frac{-q^2}{2\sigma_s^{\ 2}} - \rho \quad , \quad p = \sqrt{-q^2 - 2\sigma_s^{\ 2}(\rho+\alpha)}$$

Solving eq.(14)

$$K' - \frac{\alpha}{q} K = 0$$

$$K = de^{\frac{\alpha}{q}k} \tag{11.a}$$

$$\phi(g,k) = G(g)K(k)$$

$$= \sum_{n=1}^{\infty} A_n e^{\frac{qg}{\lambda\sigma_s^2}} \left[-\tan\frac{p\overline{g}}{\lambda\sigma_s^2}\cos\frac{p_n g}{\lambda\sigma_s^2} + \sin\frac{p_n g}{\lambda\sigma_s^2} \right] \cdot e^{\frac{\alpha_n}{q}k} \tag{11.b}$$

$$F(g,k) = \phi(g,k) + w(g)$$

Meanwhile, $w(g)$ and $\phi(g,k)$ have been solved as (8a),(8b) and (11a),(11b)

i.e. $$F(g,k) = \sum_{n=1}^{\infty} A_n e^{\frac{qg}{\lambda\sigma_s^2}} \left[-\tan\frac{p\overline{g}}{\lambda\sigma_s^2}\cos\frac{p_n g}{\lambda\sigma_s^2} + \sin\frac{p_n g}{\lambda\sigma_s^2} \right] \cdot e^{\frac{\alpha_n}{q}k} + B(\overline{g})e^{m_1(g-\overline{g})} \tag{17}$$

where $m_1 = \dfrac{q + \sqrt{q^2 + 2\sigma_s^2\rho}}{\lambda\sigma_s^2}$

Finally, substituting the initial equation, Eq.(5),into Eq.(17), then A_n can be obtained

$$F(g,0) = h(g) = \sum_{n=1}^{\infty} A_n e^{\frac{qg}{\lambda\sigma_s^2}} \left[-\tan\frac{p\overline{g}}{\lambda\sigma_s^2}\cos\frac{p_n g}{\lambda\sigma_s^2} + \sin\frac{p_n g}{\lambda\sigma_s^2} \right] + B(\overline{g})e^{m_1(g-\overline{g})}$$

$$A_n = \frac{2}{\overline{g} - (-\infty)} \int_{-\infty}^{\overline{g}} \left[h(g) - B(\overline{g})e^{m_1(g-\overline{g})} \right] \cdot e^{\frac{qg}{\lambda\sigma_s^2}} \left[-\tan\frac{p\overline{g}}{\lambda\sigma_s^2}\cos\frac{p_n g}{\lambda\sigma_s^2} + \sin\frac{p_n g}{\lambda\sigma_s^2} \right] dg$$

$$A_n = 0 \tag{18}$$

Therefore, $F(g,k) = B(\overline{g})e^{m_1(g-\overline{g})}$ (19)

Eq. (18) states that whatever the value of $h(g)$, A_n is always equal to zero. This is a significant finding because Eq. (18) makes the evaluation of optimal contingent value of OMRs a very simple task. The optimal contingent value of OMRs, taking into consideration the asymmetry payoff and remaining repurchase amount, is dependent on the maximum bound of g, $\overline{g}$, maximum value of $F(\overline{g},k)$ and the other necessary given input variables.

This solution might seem troubling in that it does not appear to depend on k. In fact, it does depend on k, through the "constant" $B(\overline{g})$. As we will see, k must be found in conjunction with the boundary $g^* = g^*(k)$, and hence will vary with k .

[Remark]: The above derivation can refer to Nagle/ Saff/Snider, 1996, Fundamentals of Differential

Equations and Boundary Value Problems CH. 11.